PCI
MANUAL FOR THE DESIGN
OF
HOLLOW CORE SLABS

SECOND EDITION

by

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Prepared for the
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INTRODUCTION

Purpose of Manual

The application and design of precast, prestressed hollow core slabs is similar to that of other prestressed members. However, there are situations which are unique to hollow core slabs either because of the way the slabs are produced or because of the application of the slabs.

For special situations, hollow core producers have developed design criteria and conducted in-house testing to verify that their approaches are valid. In fact, there is consistency between the many types of hollow core slabs available. The purpose of this manual is to bring together those things that are common, that are verified by test and that can be universally applied to hollow core slabs. Because there are differences, some topics covered will also point to the differences where closer coordination with the local producer is required.

This manual was prepared by Computerized Structural Design, S.C., Milwaukee, Wisconsin with input and direction from the PCI Hollow Core Slab Producers Committee. Additionally, the fire and acoustical sections were prepared by Armand Gustaferro of The Consulting Engineers Group, Inc., Mt. Prospect, Illinois and Allen H. Shiner of Shiner and Associates, Inc., Skokie, Illinois, respectively. All reasonable care has been used to verify the accuracy of material contained in this manual. However, the manual should be used only by those experienced in structural design and should not replace good structural engineering judgment.

Scope of Manual

This document is intended to cover the primary design requirements for hollow core floor and roof systems. In instances where the design is no different than for other prestressed members, the PCI Design Handbook and the ACI Building Code should be consulted for more in-depth discussion.

For the architect or consulting engineer, this manual is intended as a guideline for working with hollow core slabs, a guide for the use and application of hollow core slabs and an indication of some of the limitations of hollow core slabs. For the plant engineer, the manual will hopefully present some backup and reference material for dealing with everyday design problems.

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NOTATION

A = Cross-sectional area
a = Depth of equivalent compression stress block
a_θ = Depth of equivalent compression stress block under fire conditions
A_{cr} = Area of crack face
A_e = Net effective slab bearing area
A_{ps} = Area of prestressed reinforcement
A_{vf} = Area of shear friction reinforcement
b = Width of compression face
b_w = Net web width of hollow core slab
C = Confinement factor
C = Compressive force
C = Seismic factor dependent on site and structure fundamental period
C = Factor for calculating steel relaxation losses as given in Table 2.2.3.2
c = Distance from extreme compression fiber to neutral axis
CR = Prestress loss due to concrete creep
C_s = Seismic coefficient
D = Dead load
d = Distance from extreme compression fiber to centroid of non-prestressed tension reinforcement
d_b = Nominal diameter of reinforcement
d_p = Distance from extreme compression fiber to centroid of prestressed reinforcement
DW = Distribution width
e = Distance from neutral axis to centroid of prestressed reinforcement
E_c = Modulus of elasticity of concrete
E_{ci} = Modulus of elasticity of concrete at the time of initial prestress
ES = Prestress loss due to elastic shortening of concrete
E_s = Modulus of elasticity of steel reinforcement
f'_c = Specified design compressive strength of concrete
f'_{ci} = Compressive strength of concrete at the time of initial prestress
f_{cir} = Net compressive stress in concrete at centroid of prestressed reinforcement at time of initial prestress
f_{cds} = Stress in concrete at centroid of prestressed reinforcement due to superimposed dead load
f_d = Stress at extreme tension fiber due to unfactored member self weight
F_i = Portion of base shear applied at level i
f_{pc} = Compressive stress in concrete at the centroid of the section due to effective prestress for non-composite sections or due to effective prestress and moments resisted by the precast section alone for composite sections
f_{pe} = Compressive stress in concrete at extreme fiber where external loads cause tension due to the effective prestress only
f_{ps} = Stress in prestressed reinforcement at nominal strength
$f_{ps\theta}$ = Stress in prestressed reinforcement at fire strength
f'_{ps} = Maximum steel stress in partially developed strand
f_{pu} = Specified tensile strength of prestressing steel
$f_{pu\theta}$ = Tensile strength of prestressing steel at elevated temperatures
F_{px} = Force applied to diaphragm at level under consideration
f_{se} = Effective stress in prestressing steel after all losses
f_{si} = Stress in prestressing steel at initial prestress
F_t = Additional portion of base shear applied at top level
f_u = Usable grout strength in a horizontal joint
f_y = Steel yield strength
h = Overall member depth
h_n = Net height of grout in keyway between slab units
I = Occupancy importance factor
I = Cross-sectional moment of inertia
J = Factor for calculating steel relaxation losses as given in Table 2.2.3.1
k = Fraction of total load in a horizontal joint in a grout column
K_{cir} = Factor for calculating elastic shortening prestress losses
K_{cr} = Factor for calculating prestress losses due to concrete creep
K_{es} = Factor for calculating prestress losses due to elastic shortening
K_{re} = Factor for calculating prestress losses due to steel relaxation as given in Table 2.2.3.1

K_{sh} = Factor for calculating prestress losses due to concrete shrinkage
K'_u = Factor from PCI Handbook Fig. 4.12.2 for calculating flexural design strength
L = Live load
ℓ = Span length
ℓ_d = Reinforcement development length
ℓ_e = Strand embedment length from member end to point of maximum stress
ℓ_f = Flexural bond length
ℓ_t = Strand transfer length
M = Service load moment
M_{cr} = Cracking moment
M_d = Unfactored dead load moment
M_g = Unfactored self-weight moment
M_n = Nominal flexural strength
$M_{n\theta}$ = Flexural strength under fire conditions
M_{max} = Maximum factored moment due to externally applied loads
= $M_u - M_d$
M_{sd} = Unfactored moment due to superimposed dead load
M_u = Factored design moment
M_θ = Applied fire moment
P = Effective force in prestressing steel after all losses
P_o = Effective prestress force at release prior to long term losses
P_i = Initial prestress force after seating losses
Q = First moment of area
R = Fire endurance rating
RE = Prestress loss due to steel relaxation
R_e = Reduction factor for load eccentricity in horizontal joints
RH = Ambient relative humidity
R_w = Seismic coefficient dependent on structural system type
S = Section modulus
SH = Prestress loss due to concrete shrinkage
T = Tensile force
t_g = Width of grout column in horizontal joint
V = Seismic base shear
V_c = Nominal shear strength of concrete
V_{ci} = Nominal shear strength of concrete in a shear-flexure failure mode
V_{cw} = Nominal shear strength of concrete in a web shear failure mode
V_d = Shear due to unfactored self weight
V_h = Horizontal beam shear
V_i = Factored shear force due to externally applied loads occurring simultaneously with M_{max}
= $V_u - V_d$
V_n = Nominal shear strength of a member
V_s = Nominal shear strength provided by shear reinforcement
V_u = Design shear force
V/S = Volume to surface ratio
w = Uniformly distributed load
w = Bearing area length
W = Total dead load plus other applicable loads for seismic design
w_i = Portion of W at level i
w_{px} = Portion of W at level under consideration
y_b = Distance from neutral axis to extreme bottom fiber
y_t = Used as either distance to top fiber or tension fiber from neutral axis
Z = Seismic zone factor
β_1 = Factor defined in ACI 318-95, Section 10.2.7.3
γ_p = Factor for type of prestressing strand
δ_{all} = Limiting free end slip
δ_s = Actual free end slip
ε_{ps} = Strain in prestressed reinforcement at nominal flexural strength
ε_s = Strain in prestressed reinforcement
ε_{se} = Strain in prestressed reinforcement after losses
μ = Shear friction coefficient
μ_e = Effective shear friction coefficient
ρ_p = Ratio of prestressed reinforcement
ρ' = Ratio of compression reinforcement
ϕ = ACI strength reduction factor
ω = $\rho f_y/f'_c$
ω' = $\rho' f_y/f'_c$
ω_p = $\rho_p f_{ps}/f'_c$
ω_w = Reinforcement index for flanged sections
ω'_w = Reinforcement index for flanged sections
ω_{pw} = Reinforcement index for flanged sections
ω_{pu} = $\rho_p f_{pu}/f'_c$
θ = Subscript denoting fire conditions

HOLLOW CORE SLAB SYSTEMS

1.1 Methods of Manufacturing

A hollow core slab is a precast, prestressed concrete member with continuous voids provided to reduce weight and, therefore, cost and, as a side benefit, to use for concealed electrical or mechanical runs. Primarily used as floor or roof deck systems, hollow core slabs also have applications as wall panels, spandrel members and bridge deck units.

An understanding of the methods used to manufacture hollow core slabs will aid in the special considerations sometimes required in the use of hollow core slabs. Hollow core slabs are cast using various methods in the seven major systems available today. Because each production system is patented, producers are usually set up on a franchise or license basis using the background, knowledge and expertise provided with the machine development. Each producer then has the technical support of a large network of associated producers.

Two basic manufacturing methods are currently in use for the production of hollow core slabs. One is a dry cast or extrusion system where a very low slump concrete is forced through the machine. The cores are formed with augers or tubes with the concrete being compacted around the cores. The second system uses a higher slump concrete. Sides are formed either with stationary, fixed forms or with forms attached to the machine with the sides being slip formed. The cores in the normal slump, or wet cast, systems are formed with either lightweight aggregate fed through tubes attached to the casting machine, pneumatic tubes anchored in a fixed form or long tubes attached to the casting machine which slip form the cores.

Table 1.1 lists the seven major hollow core systems available today along with the basic information on the casting technique. Various names may be used by local licensees to describe the same products. In most cases, the slabs are cast on long line beds, normally 300 ft to 600 ft long. Slabs are then sawcut to the appropriate length for the intended project.

The economy of the generalized hollow core system is in the quantity of slabs that can be produced at a given time with a minimum of labor required. Each slab on a given casting line will have the same number of prestressing strands. Therefore, the greatest production efficiency is obtained by mixing slabs with the same reinforcing requirements from several projects on a single production line. This implies that best efficiency for a single project is obtained if slab requirements are repetitive.

1.2 Materials

As stated previously, hollow core slabs are produced with two basic concrete mixes; low slump and normal slump concrete. For the low slump concretes, water content is limited to slightly more than that required for cement hydration. Water-cement ratios are typically about 0.3. Mixing is critical because the limited water available must be well dispersed in the mix. Water reducing admixtures can be used to optimize a mix by reducing cement and water requirements while still retaining adequate workability for proper compaction of the concrete by the machine. Air entrainment admixtures are not effective in the dry mix concrete. With the low water-cement ratios and compaction placing method, air is difficult to disperse well and maintain.

Table 1.1 Hollow Core Systems

Manufacturer	Machine Type	Concrete Type/Slump	Core Form
Dy-Core	Extruder	Dry/Low	Tubes
Dynaspan	Slip Form	Wet/Normal	Tubes
Elematic	Extruder	Dry/Low	Auger/Tube
Flexicore	Fixed Form	Wet/Normal	Pneumatic Tubes
Spancrete	Slip Form	Dry/Low	Tubes
SpanDeck	Slip Form	Wet/Normal	Filler aggregate
Ultra-Span	Extruder	Dry/Low	Augers

The wet cast products (those cast with normal slump concrete), have water-cement ratios in the range of 0.4 to 0.45. Depending on the slip forming system used, slumps of 2 to 5 inches (50 - 130 mm) are used. The mix design and use of admixtures is dependent on achieving a mix that will hold its shape consistent with the forming technique used.

Latex feathering ready for direct carpet application

Aggregates vary in the various manufacturing processes depending on what type is locally available. Maximum aggregate size larger than pea gravel is rarely used because of the confined areas into which concrete must be placed. Light weight aggregates are occasionally used to reduce the weight of the sections and to achieve a significant reduction in required equivalent thickness in a fire rated application. Concrete unit weights ranging from 110 to 150 pcf (1760 - 2400 kg/m^3) are used in the industry.

Strand use in hollow core slabs includes about every size and type of strand produced depending on what is available to a particular producer. The trend is toward primary use of the larger $^1/_2$ in (13 mm) diameter, low relaxation strand. The philosophy of strand use varies from using many strand sizes to optimize cost for a given project to using only one or two strand sizes for simplicity of inventory and production.

Acoustical spray on exposed slab ceiling

Except for special situations, keyway grout is normally a sand and Portland cement mixture in proportions of about 3:1. The amount of water used is a function of the method used to place the grout but will generally result in a wet mix so keyways may be easily filled. Shrinkage cracks may occur in the keyways, but configuration of the key is such that vertical load transfer can still occur with the presence of a shrinkage crack. Rarely is grout strength required in excess of 2000 psi (13.8 MPa) for vertical load transfer.

Although it is discouraged, non-shrink, non-staining grout is occasionally specified for use in keyways. In evaluating the potential benefits of non-shrink grout, the volume of grout must be compared to the overall volume of concrete in the slabs and support materials. Because the size of the keyway is small in relation to a floor or roof assembly of slabs, total shrinkage will be affected only to a minor degree. Shrinkage cracks can still

Electrical and HVAC application

occur in the keyways and there is little benefit to be gained in comparison with the additional cost.

1.3 Advantages of Hollow Core Slabs

Hollow core slabs are most widely known for providing economical, efficient floor and roof systems. The top surface can be prepared for the installation of a floor covering by feathering the joints with a latex cement, installing non-structural fill concretes ranging from $^{1}/_{2}$ in to 2 in (13 - 51 mm) thick depending on the material used, or by casting a composite structural concrete topping. The underside can be used as a finished ceiling as installed, by painting, or by applying an acoustical spray.

When properly coordinated for alignment, the voids in a hollow core slab may be used for electrical or mechanical runs. For example, routing of a lighting circuit through the cores can allow fixtures in an exposed slab ceiling without unsightly surface mounted conduit. Slabs used as the heated mass in a passive solar application can be detailed to distribute the heated air through the cores.

Structurally, a hollow core slab provides the efficiency of a prestressed member for load capacity, span range, and deflection control. In addition, a basic diaphragm is provided for resisting lateral loads by the grouted slab assembly provided proper connections and details exist. A detailed discussion of diaphragm capabilities is presented in Chapter 4.

Excellent fire resistance is another attribute of the hollow core slab. Depending on thickness and strand cover, ratings up to a 4 hour endurance can be achieved. A fire rating is dependent on equivalent thickness for heat transmission, concrete cover over the prestressing strands for strength in a high temperature condition, and end restraint. Underwriters Laboratories publishes fire ratings for various assemblies. However, many building codes allow a rational design procedure for strength in a fire. This procedure, described in detail in Chapter 6, considers strand temperature in calculating strength. Required fire ratings should be clearly specified in the contract documents. Also, the fire rating should be considered in determining the slab thickness to be used in preliminary design.

Used as floor-ceiling assemblies, hollow core slabs have the excellent sound transmission characteristics associated with concrete. The Sound Transmission Class rating ranges from about 47 to 57 without topping and the Impact Insulation Class rating starts at about 23 for a plain slab and may be increased to over 70 with the addition of carpeting and padding. Detailed information on the acoustical properties of hollow core slabs is presented in Chapter 7.

1.4 Framing Concepts

The primary consideration in developing a framing scheme using hollow core slabs is the span length. For a given loading and fire endurance rating, span length and slab thickness may be optimized by consulting a producer's published load tables. Section 1.7 presents sample load tables and instructions for the use of the tables. The PCI Design Handbook[1] recommends limits on span-depth ratios for the hollow core slabs. For roof slabs, a span-depth ratio limit of 50 is suggested and for floor slabs, a limit of 40 is suggested. In practice, a span-depth ratio of 45 is common for floors and roofs when fire endurance, openings, or heavy or sustained live loads do not control a design.

Consideration must be given to factors which affect slab thickness selection for a given span. Heavy superimposed loads, as required by the function of a system, would require a lower span-depth ratio. Similarly, heavy partitions or a large number of openings will result in higher load capacity requirements. The fire resistance rating required for the application will also affect the load capacity of a slab. As the code required fire rating increases, prestressing strands can be raised for more protection from the heat. The smaller effective strand depth will result in a lower load capacity. Alternatively, a rational design procedure can be used to consider the elevated strand temperatures during a fire. This fire design condition may control a slab design and, again, result in a lower load capacity.

Once slab thicknesses and spans are selected, the economics of layout become important. While ends cut at an angle can be designed and supplied, it is most efficient to have the bearing perpendicular to the span so square cut ends can be used.

It is also desirable to have the plan dimensions fit the slab module. This is dependent upon the slab systems available in the project area. Non-module plan dimensions can be accommodated using partial width slabs. Some producers intentionally cast narrow widths as filler pieces while others use a section split from a full slab. Such a split section might be created by a longitudinal sawcut or a break if the edge will not be exposed to view.

Construction tolerances must be accounted for in developing a plan layout. Tolerance on slab length may be taken up by allowing a gap at the slab ends in the bearing detail. On the non-bearing sides, clearance may be provided by using a detail where the slabs lap over a wall or beam. If the slab edge butts a wall or beam, a gap should be provided. Refer to local producers' information for recommendations of proper tolerances.

When a hollow core slab deck is exposed to weather for a long period of time during construction, water can accumulate in the cores. The primary source of water infiltration is at the butt joints. In cold weather, this water can freeze and expand causing localized damage. One remedy for this situation is to drill weep holes at the slab ends under each core. The need for such weep holes is generally known only after a construction schedule is established. The specifier and the slab supplier are not usually in a position to know of such a need in advance.

Hollow core members will be cambered as with any other prestressed flexural member. In the planning stages, consideration should be given to the causes of differential camber. For two slabs of identical length and prestressing, the camber may be different because of concrete and curing variations. This factor is independent of a framing scheme. However, joints between slabs of unequal spans or joints at which a change in the span direction occurs, will cause a potential differential camber problem. This must be recognized and dealt with in the design layout. Wall locations may hide such a joint, but the door swing might be directed to the least variable side.

Camber must also be accommodated when a topping is to be provided. The quantity of topping required must consider the amount of camber and the function of the floor. In occupancies where flat floors are not a requirement, a constant topping thickness may be used to follow the curvature of the slabs. At the other extreme, if a "flat" floor is required in a structure consisting of multiple bays of varying length and change in slab direction, the highest point will determine the top elevation of the topping. A greater amount of topping will then be required in "low" areas. These considerations must be dealt with in the planning stages to both control costs and minimize questions and potential for "extras" during construction.

Camber, camber growth, and deflections must be considered when slabs run parallel to a stiff vertical element such as a wall (e.g. slabs running parallel to the front wall of an elevator). The door rough opening should allow for camber to produce proper door installation. Alternatively, the slab span might be rearranged so the front wall is a bearing wall. Then door problems would be alleviated.

Camber, camber growth, and deflections must be taken into account in roofing details. Where changes in relative slab position can occur, counterflashings are suggested to accommodate such changes.

1.5 Wall Panel Applications

Some hollow core slab systems can also provide slabs to be used as walls. Long line manufacturing can result in economical cladding or load bearing panels used in manufacturing or commercial applications. The hollow core wall panels are prestressed with two layers of strands for accommodating handling, structural loadings and bowing considerations. Some manufacturers can add 2 in to 4 in (51 - 102 mm) of insulation to the hollow core section with a 1 $^1/_2$ in thick to 3 in (38 - 76 mm) thick concrete facing to create an insulated sandwich panel.

A variety of architectural finishes are available with hollow core wall panels. While the finishes can be very good, the variety of finishes available is different from those typically available with true architectural precast concrete panels. In judging the quality of finish on hollow core wall panels, consideration must be given to the manufacturing process.

1.6 Design Responsibilities

It is customary in the hollow core industry for the producer to perform the final engineering for the product to be supplied to the job. This would include design for vertical loads and lateral loads specified by the Engineer of Record, embedded items for specified connection forces, and handling and shipping. However, the Engineer of Record plays a very important role in the design process. Prior to selection of the hollow core producer, enough preliminary planning should be done to insure that the specified floor and roof system is achievable. That is, the project should be one that can be engineered without requiring changes from the contract documents.

The contract documents must clearly indicate design criteria to which hollow core slabs will have to conform. This is especially important when the hollow core slabs must interface with other construction materials. When connections are required, the forces to be transmitted through the connections must be specified in the contract documents. The producer is best able to determine the most efficient connection element to be embedded in the slab. However, the balance of a connection which interfaces with another material should be detailed in the contract documents.

The Engineer of Record also has a responsibility in the review and approval of erection drawings prepared by the precast producer. Review of these drawings is the last opportunity to assure that the producer's understanding of the project coincides with the intent of design. Erection drawings should be checked for proper design loads, proper details and bearing conditions, conformance with specified fire ratings, and the location of openings.

1.7 Cross-Sections and Load Tables

Each of the major hollow core slab systems has a standard set of cross-sections that can be produced by their equipment. Available in thicknesses ranging from 4 in to 15 in (102 - 380 mm), core configurations make each system unique. Each individual producer has additional production practices which may affect the capabilities of their product. Therefore, most producers prepare and distribute load tables in their market area.

Producer load tables define the allowable live load that a given slab can safely support in addition to the slab self weight. The load capacity will be a function of the slab thickness, the amount of prestressing provided, and the location of the prestressing strands. Fire rated slabs may require additional concrete cover below the strands which will affect the load capacity.

The design criteria used to develop these load tables is defined by the ACI Building Code[2] as outlined in Chapter 2. Depending on the design criteria controlling a slab's load capacity, some advantage may be gained by understanding that in most applications, superimposed loads will consist of both dead and live loads. Where ultimate strength controls, an equivalent live load can be used to enter a load table. It is calculated as:

$$w_{equivalent} = \frac{1.4}{1.7}\text{ superimposed Dead load} + \text{Live load}$$

However, if bottom fiber tensile stresses control, no adjustment in superimposed loads may be used.

Similarly, many loading conditions consist of loads other than uniform loads. For preliminary design only, an equivalent uniform load may be calculated from the maximum moment caused by the actual loads.

$$w_{equivalent} = \frac{8\ M_{superimposed}}{\ell^2}$$

Shear will not be properly addressed in this situation. Thus, the final design must consider the actual load pattern.

Because of the uniqueness of each hollow core slab system and the many possibilities of strand patterns available from various producers, a generic hollow core slab has been developed to demonstrate design procedures. Figure 1.7.1 depicts the slab section and properties and illustrates a typical form for a producer's load tables. Throughout this manual, this section will be used to demonstrate various calculation procedures where any one of the proprietary cross-sections could be substituted. *It must be emphasized that this cross-section is not available for use and should not be specified.*

Figures 1.7.2 through 1.7.8 present the proprietary slab cross-sections currently available. The section properties are as provided by the manufacturers, but weights are based on 150 pcf (2400 kg/m^3) concrete. The actual weights may vary slightly from those given. The availability of any particular section in a given area must be verified with the local producers. Figures 1.7.9 present charts of the general range of load capacities available in a given slab thickness. As with any chart of this nature, the chart should be carefully approached and verified with local producer load tables, especially for the longest and shortest and lightest and heaviest conditions. Special care is also required when fire rated slabs must be used on a project. (See Chapter 6)

The following examples demonstrate the ways in which load tables may be used.

Example 1.7.1 Equivalent Uniform Load

From the load table in Figure 1.7.1 select a strand pattern to carry a uniform superimposed dead load of 20 psf and a uniform live load of 60 psf on a 24 foot span.

$w_{total} = 20 + 60 = 80$ psf

4-7/16 in dia. strands required: capacity = 118 psf flexural strength controls

$$w_{equivalent} = \frac{1.4}{1.7}(20) + 60 = 77 \text{ psf}$$

Use 4-3/8 in dia. strands: capacity = 79 psf flexural strength controls.

Example 1.7.2 Non-Uniform Loads

From the load table in Figure 1.7.1 select a strand pattern to carry a superimposed uniform load of 20 psf dead plus 40 psf live and a continuous wall load of 600 plf located perpendicular to the span and at midspan. The design span is 25 feet.

For preliminary design

$$M_{superimposed} = \frac{25^2}{8}(20 + 40) + \frac{25}{4}(600)$$

$$= 8438 \text{ ft-\#/ft}$$

$$w_{equivalent} = \frac{8(8438)}{25^2}$$

$$= 108 \text{ psf}$$

Try 6-3/8 in dia. strands - capacity = 120 psf

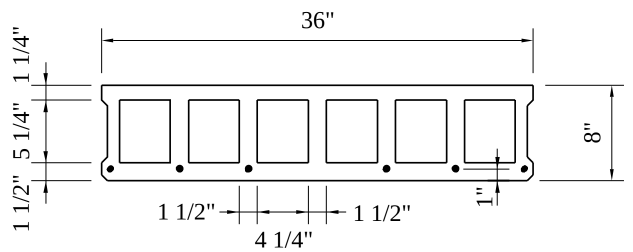
For final design use the methods of Chapter 2 particularly to check shear.

1.8 Tolerances[3]

Figure 1.8.1 shows the dimensional tolerances for precast hollow core slabs. These tolerances are guidelines only and each project must be considered individually to ensure that the tolerances shown are applicable.

Figure 1.8.2 shows erection tolerances for hollow core slabs. When establishing tolerances, the function of the slabs should be considered. For example, slabs covered by finish materials may not need the close tolerances required for exposed slabs.

Fig. 1.7.1 Generic hollow core slab



Section Properties
A = 154 in^2
I = 1224.5 in^4
b_w = 10.5 in
y_b = 3.89 in
S_b = 314.8 in^3
S_t = 297.9 in^3
wt = 53.5 psf

SAMPLE LOAD TABLE[3]

Allowable Superimposed Live Loads, psf

Strands, 270LR	φMn, ft-k	Spans, ft									
		14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**
4-3/8″	45.1	317	270	232	200	174	152	133	116	102	90
6-3/8″	65.4			356	311	272	240	212	188	168	150
4-7/16″	59.4			320	278	243	214	189	167	148	132
6-7/16″	85.0					343[1]	311[1]	283[1]	258	231	208
4-1/2″	76.7					327	289	257	229	204	183
6-1/2″	105.3							317[1]	290[1]	267[1]	247[1]

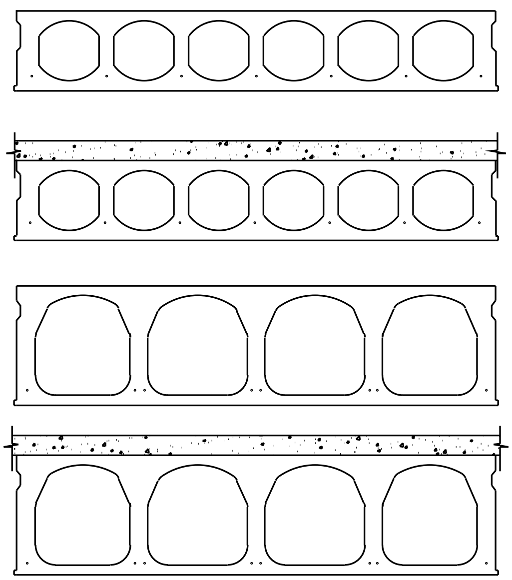
Strands, 270LR	φMn, ft-k	**24**	**25**	**26**	**27**	**28**	**29**	**30**
4-3/8″	45.1	79	79	69	61	53	46	
6-3/8″	65.4	134	120	108	97	87	78	70
4-7/16″	59.4	118	105	94	84	75	67	59
6-7/16″	85.0	187	169	153	139	126	114	104
4-1/2″	76.7	165	148	134	121	109	99	90
6-1/2″	105.3	227[1]	210[1]	195[2]	178[2]	163[2]	149[2]	137[2]

1 - Values are governed by shear strength.
2 - Values are governed by allowable tension
3 - Table based on 5000 psi concrete with $6\sqrt{f'_c}$ allowable tension. Unless noted, values are governed by strength design.

Note: *This slab is for illustration purposes only. Do not specify this slab for a project.*

Fig. 1.7.2

Trade name: Dy-Core
Equipment Manufacturer: Mixer Systems, Inc., Pewaukee, Wisconsin

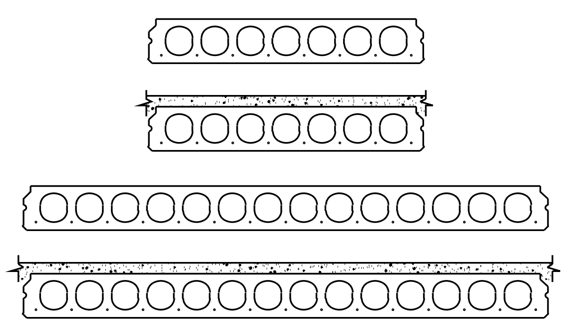


Section	Untopped				with 2″ topping		
width x depth	A in^2	y_b in	I in^4	wt psf	y_b in	I in^4	wt psf
4′-0″ x 6″	142	3.05	661	37	4.45	1475	62
4′-0″ x 8″	193	3.97	1581	50	5.43	3017	75
4′-0″ x 10″	215	5.40	2783	56	6.89	4614	81
4′-0″ x 12″	264	6.37	4773	69	7.89	7313	94
4′-0″ x 15″	289	7.37	8604	76	9.21	13225	101

Note: All sections not available from all producers. Check availability with local manufacturers.

Fig. 1.7.3

Trade name: Dynaspan®
Equipment Manufacturer: Dynamold Corporation, Salina, Kansas

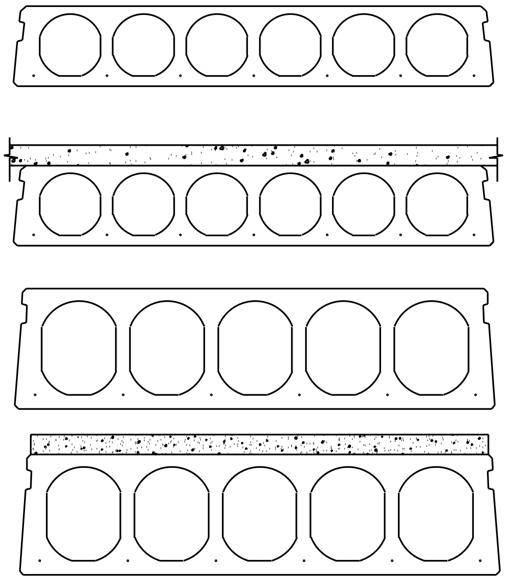


Section	Untopped				with 2″ topping		
width x depth	A in^2	y_b in	I in^4	wt psf	y_b in	I in^4	wt psf
4′-0″ x 4″	133	2.00	235	35	3.08	689	60
4′-0″ x 6″	165	3.02	706	43	4.25	1543	68
4′-0″ x 8″	233	3.93	1731	61	5.16	3205	86
4′-0″ x 10″	260	4.91	3145	68	6.26	5314	93
8′-0″ x 6″	338	3.05	1445	44	4.26	3106	69
8′-0″ x 8″	470	3.96	3525	61	5.17	6444	86
8′-0″ x 10″	532	4.96	6422	69	6.28	10712	94
8′-0″ x 12″	615	5.95	10505	80	7.32	16507	105

Note: All sections not available from all producers. Check availability with local manufacturers.

Fig. 1.7.4

Trade name: Elematic®
Equipment Manufacturer: Mixer Systems, Inc., Pewaukee, Wisconsin

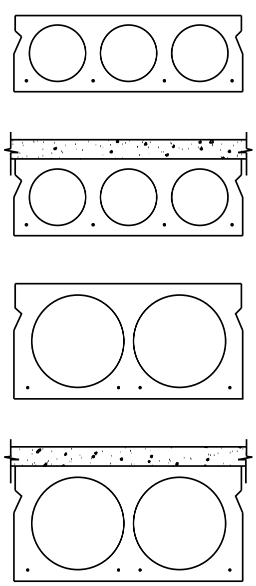


Section	Untopped				with 2″ topping		
width x depth	A in^2	y_b in	I in^4	wt psf	y_b in	I in^4	wt psf
4′-0″ x 6″	157	3.00	694	41	4.33	1557	66
4′-0″ x 8″	196	3.97	1580	51	5.41	3024	76
4′-0″ x 10″(5)	238	5.00	3042	62	6.49	5190	87
4′-0″ x 10″(6)	249	5.00	3108	65	6.44	5280	90
4′-0″ x 12″	274	6.00	5121	71	7.56	8134	96

Note: Elematic is also availble in 96″ width. All sections not available from all producers. Check availability with local manufacturers.

Fig. 1.7.5

Trade name: Flexicore®
Licensing Organization: The Flexicore Co. Inc., Dayton, Ohio

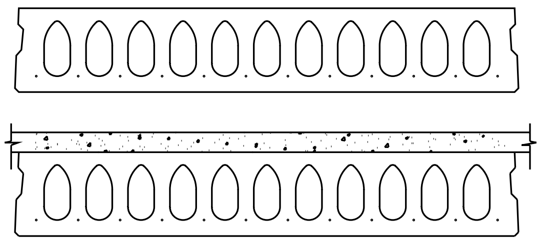


Section	Untopped				with 2″ topping		
width x depth	A in^2	y_b in	I in^4	wt psf	y_b in	I in^4	wt psf
1′-4″ x 6″	55	3.00	243	43	4.23	523	68
2′-0″ x 6″	86	3.00	366	45	4.20	793	70
1′-4″ x 8″	73	4.00	560	57	5.26	1028	82
2′-0″ x 8″	110	4.00	843	57	5.26	1547	82
1′-8″ x 10″	98	5.00	1254	61	6.43	2109	86
2′-0″ x 10″	138	5.00	1587	72	6.27	2651	97
2′-0″ x 12″	141	6.00	2595	73	7.46	4049	98

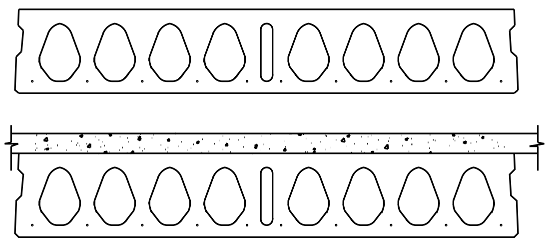
Note: All sections not available from all producers. Check availability with local manufacturers.

Fig. 1.7.6

Trade name: Spancrete®
Licensing Organization: Spancrete Machinery Corp., Milwaukee, Wisconsin



Ultralight Spancrete®



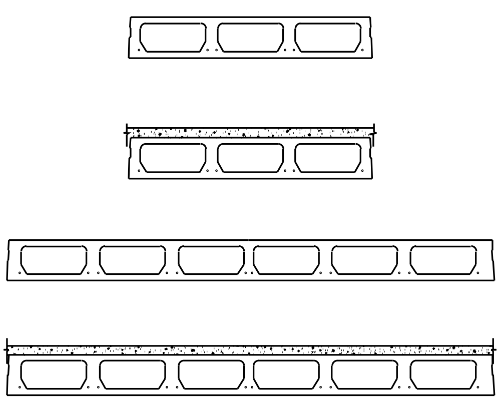
Section	Untopped				with 2″ topping		
width x depth	A in^2	y_b in	I in^4	wt psf	y_b in	I in^4	wt psf
4′-0″ x 4″	138	2.00	238	34	3.14	739	59
4′-0″ x 6″	189	2.93	762	46	4.19	1760	71
4′-0″ x 8″	258	3.98	1806	63	5.22	3443	88
4′-0″ x 10″	312	5.16	3484	76	6.41	5787	101
4′-0″ x 12″	355	6.28	5784	86	7.58	8904	111
4′-0″ x 15″	370	7.87	9765	90	9.39	14351	115

4′-0″ x 8″	246	4.17	1730	60	5.41	3230	85
4′-0″ x 10″	277	5.22	3178	67	6.58	5376	92
4′-0″ x 12″	316	6.22	5311	77	7.66	8410	102

Note: Spancrete is also available in 40″ and 96″ widths. All sections are not available from all producers. Check availability with local manufacturer.

Fig. 1.7.7

Trade name: SpanDeck®
Licensing Organization: Fabcon, Incorporated, Savage, Minnesota

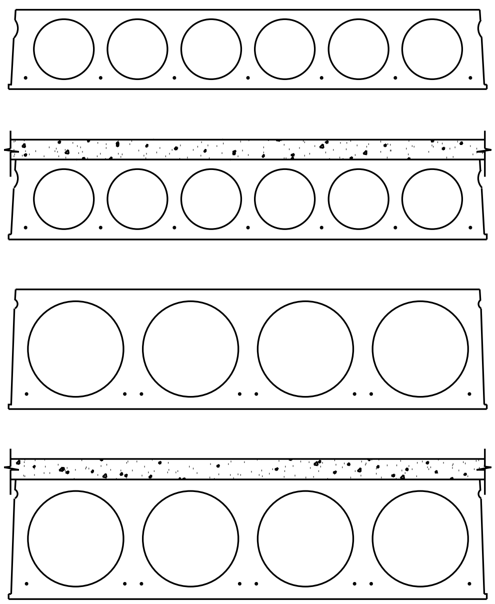


Section	Untopped				with 2″ topping		
width x depth	A in^2	y_b in	I in^4	wt psf	y_b in	I in^4	wt psf
4′-0″ x 8″	246	3.75	1615	62	5.55	2791	87
4′-0″ x 12″	298	5.87	5452	75	8.01	7856	100
8′-0″ x 8″	477	3.73	3236	60	5.53	5643	85
8′-0″ x 12″	578	5.86	10909	72	7.98	15709	97

Note: All sections not available from all producers. Check availability with local manufacturers.

Fig. 1.7.8

Trade name: Ultra-Span
Licensing Organization: Ultra-Span Technologies, Inc., Winnipeg, Manitoba, Canada



Section	Untopped				with 2″ topping		
width x depth	A in^2	y_b in	I in^4	wt psf	y_b in	I in^4	wt psf
4′-0″ x 4″	154	2.00	247	40	2.98	723	65
4′-0″ x 6″	188	3.00	764	49	4.13	1641	74
4′-0″ x 8″	214	4.00	1666	56	5.29	3070	81
4′-0″ x 10″	259	5.00	3223	67	6.34	5328	92
4′-0″ x 12″	289	6.00	5272	75	7.43	8195	100

Note: All sections are not available from all producers. Check availability with local manufacturers.

Fig. 1.7.9(a) Slab load ranges

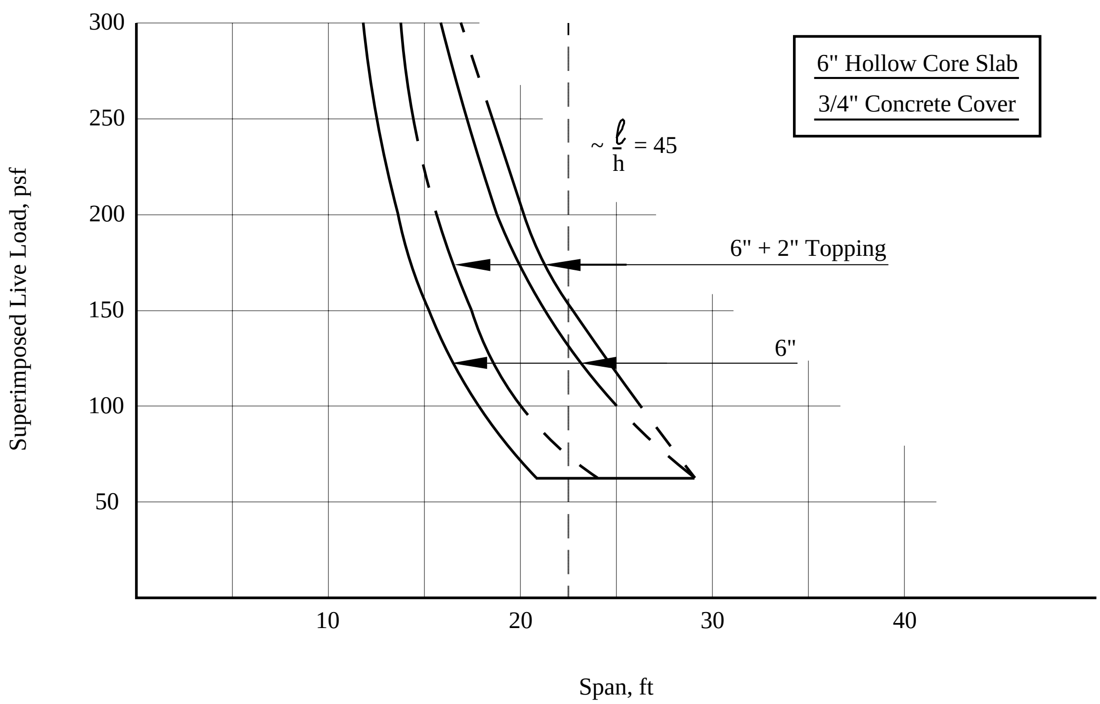


Fig. 1.7.9 (b) Slab load ranges

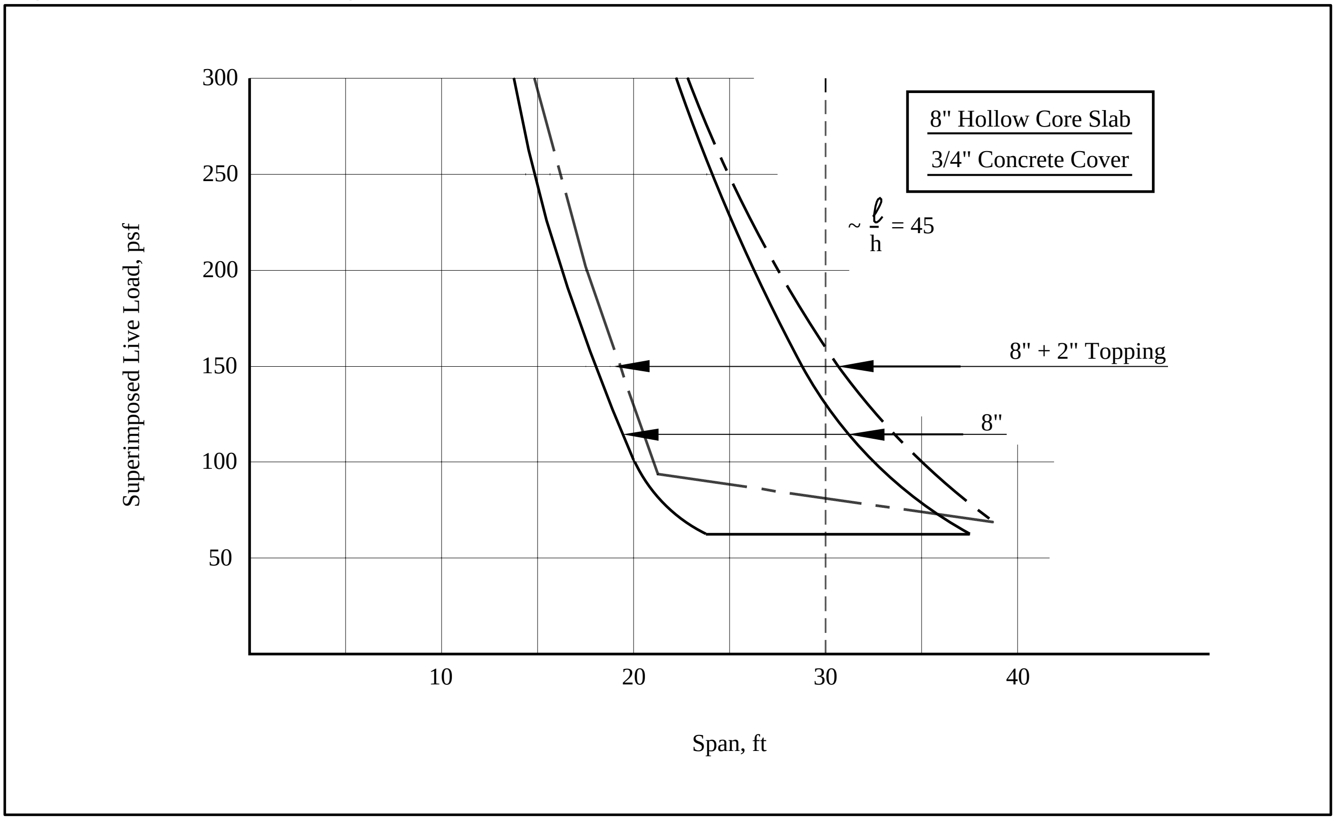


Fig. 1.7.9(c) Slab load ranges

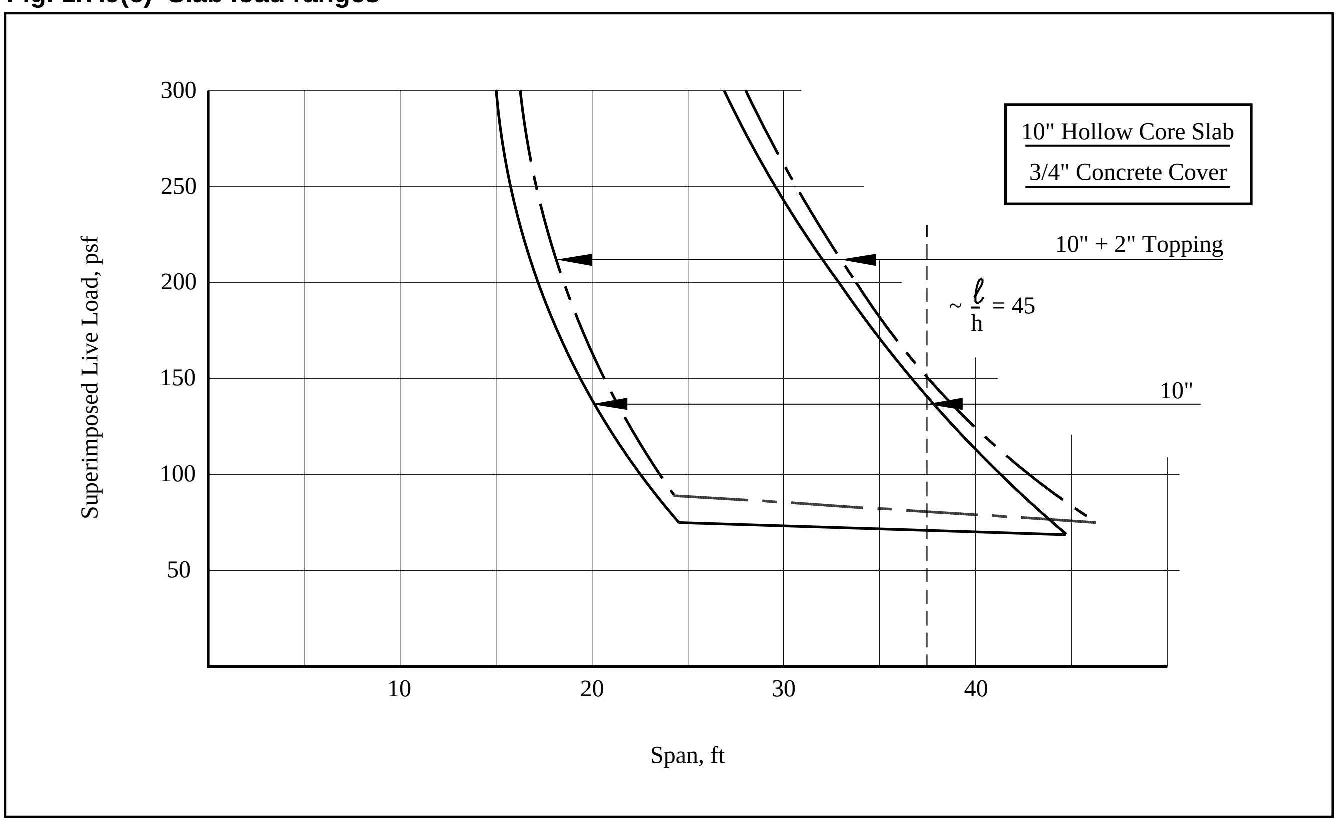


Fig. 1.7.9 (d) Slab load ranges

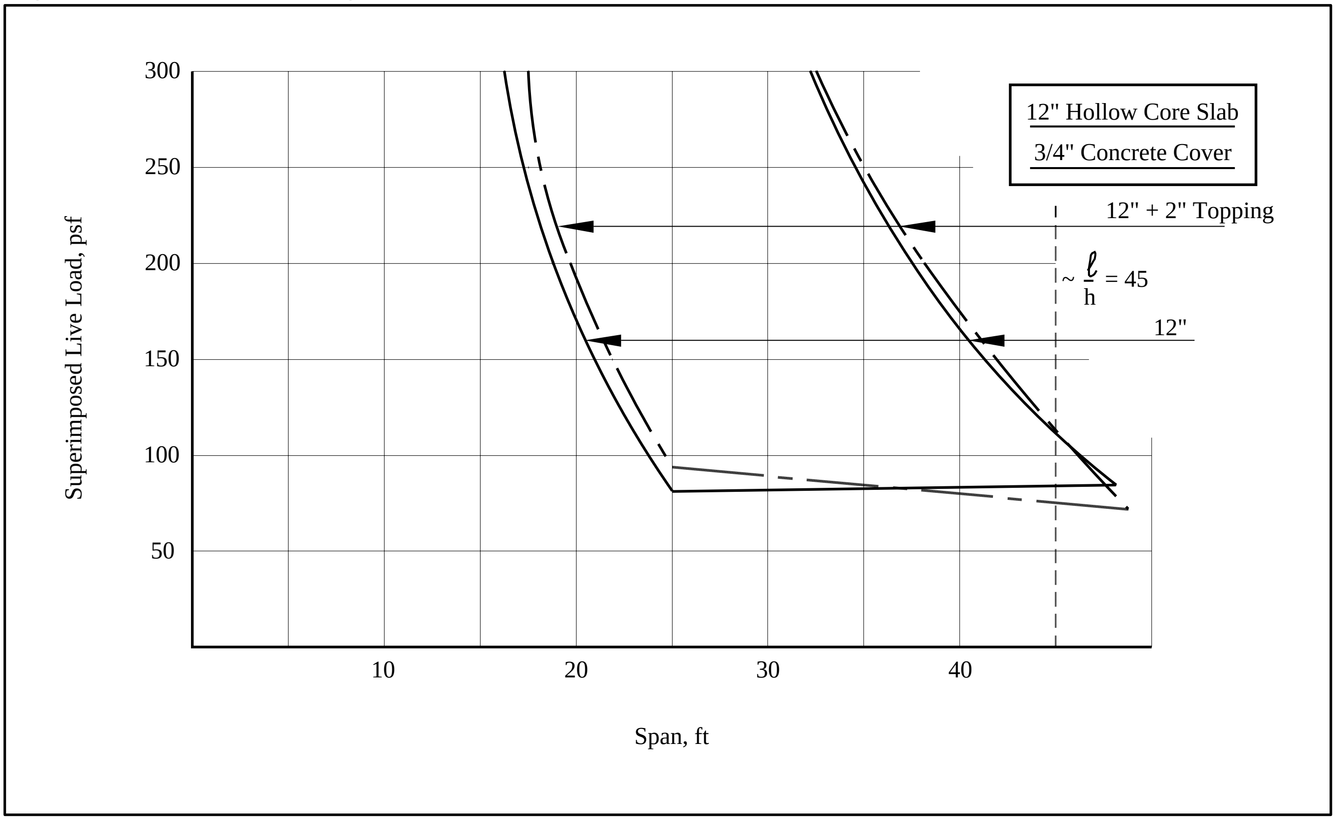


Fig. 1.7.9(e) Slab load ranges

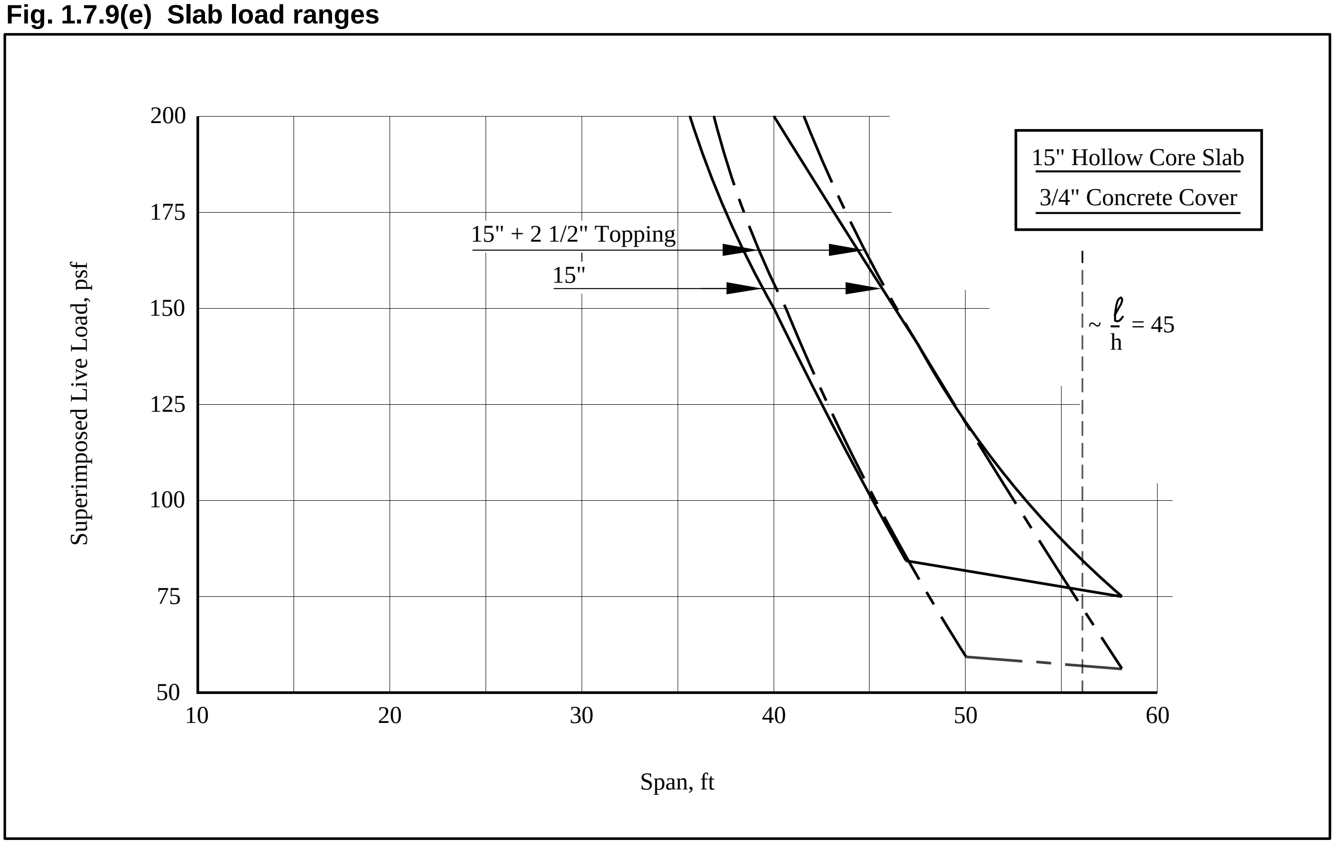


Fig. 1.8.1 Product tolerances – hollow core slabs

a = Length ... $\pm^1/_2$ in
b = Width ... $\pm^1/_4$ in
c = Depth ... $\pm^1/_4$ in
d_t = Top flange thickness
Top flange area defined by the actual measured values of average d_t x b shall not be less than 85% of the nominal area calculated by d_t nominal x b nominal.
d_b = Bottom flange thickness
Bottom flange area defined by the actual measured values of average d_b x b shall not be less than 85% of the nominal area calculated by d_b nominal x b nominal.
e = Web thickness
The total cumulative web thickness defined by the actual measured value Σe shall not be less than 85% of the nominal cumulative width calculated by Σe nominal.
f = Blockout location ... ±2 in
g = Flange angle ... $^1/_8$ in per 12 in, $^1/_2$ in max.
h = Variation from specified end squareness or skew ... $\pm^1/_2$ in
i = Sweep (variation from straight line parallel to centerline of member) ... $\pm^3/_8$ in
j = Center of gravity of strand group
The CG of the strand group relative to the top of the plank shall be within $\pm^1/_4$ in of the nominal strand group CG. The position of any individual strand shall be within $\pm^1/_2$ in of nominal vertical position and $\pm^3/_4$ in of nominal horizontal position and shall have a minimum cover of $^3/_4$ in.
k = Position of plates ... ±2 in
l = Tipping and flushness of plates ... $\pm^1/_4$ in
m = Local smoothness ... $\pm^1/_4$ in in 10 ft
(does not apply to top deck surface left rough to receive a topping or to visually concealed surfaces)
Plank weight
Excess concrete material in the plank internal features is within tolerance as long as the measured weight of the individual plank does not exceed 110% of the nominal published unit weight used in the load capacity calculation.
n = Applications requiring close control of differential camber between adjacent members of the same design should be discussed in detail with the producer to determine applicable tolerances.

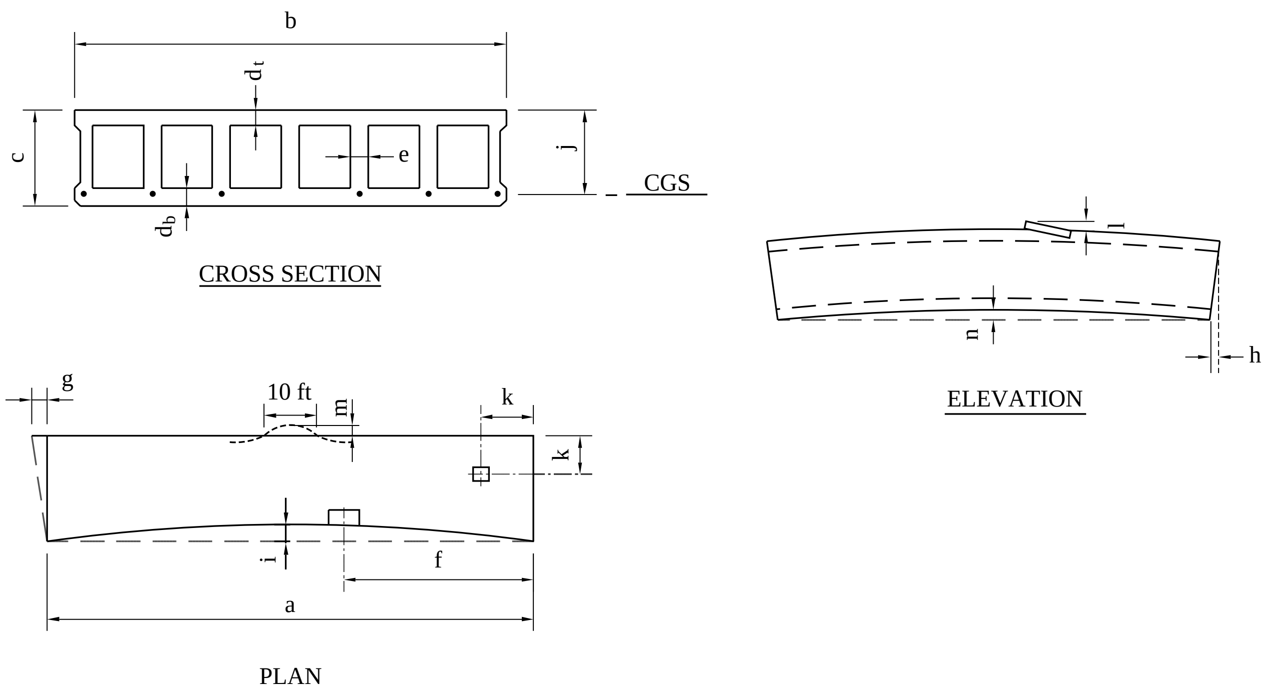


Fig. 1.8.2 Erection tolerances - hollow core floor and roof members

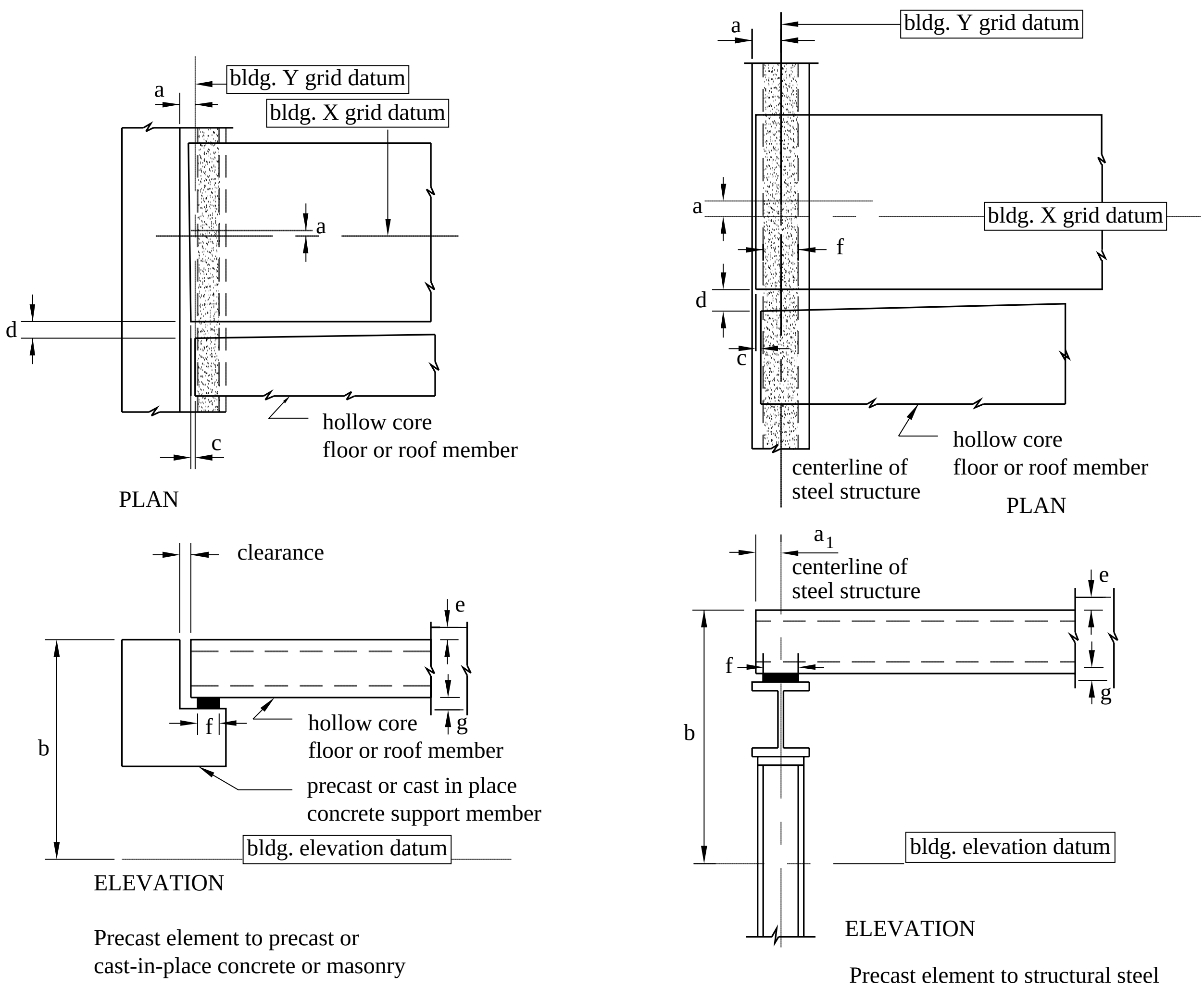
a	=	Plan location from building grid datum	± 1 in
a_1	=	Plan location from centerline of steel*	± 1 in
b	=	Top elevation from nominal top elevation at member ends	
		Covered with topping	± $^3/_4$ in
		Untopped floor	± $^1/_4$ in
		Untopped roof	± $^3/_4$ in
c	=	Maximum jog in alignment of matching edges (both topped and untopped construction)	1 in
d	=	Joint width	
		0 to 40 ft member length	± $^1/_2$ in
		41 to 60 ft member length	± $^3/_4$ in
		61 ft plus	± 1 in
e	=	Differential top elevation as erected	
		Covered with topping	$^3/_4$ in
		Untopped floor	$^1/_4$ in
		Untopped roof**	$^3/_4$ in
f	=	Bearing length*** (span direction)	± $^3/_4$ in
g	=	Differential bottom elevation of exposed hollow-core slabs****	$^1/_4$ in

* For precast concrete erected on a steel frame building, this tolerance takes precedence over tolerance on dimension "a".

** It may be necessary to feather the edges to ± $^1/_4$ in to properly apply some roof membranes.

*** This is a setting tolerance and should not be confused with structural performance requirements set by the architect/engineer.

**** Untopped installation will require a larger tolerance here.



DESIGN OF HOLLOW CORE SLABS

2.1 General

The design of hollow core slabs is governed by the ACI (318-95) Building Code Requirements for Structural Concrete.[2] As with prestressed concrete members in general, hollow core slabs are checked for prestress transfer stresses, handling stresses, service load stresses, deflections and design (ultimate) strength in shear and bending. For uniform load cases, the manufacturer's load tables will take into account these various design considerations and print a load capacity based on the governing criteria. For loading conditions other than uniform, or for the development of load tables, the design steps presented in this section are used.

An excellent reference for prestressed member design exists in the PCI Design Handbook.[1] Charts and tables provide design aids to shorten the calculation procedures. Another excellent source for design information is the PCI Standard Design Practice[4] which reflects design practices in the industry.

The generic slab presented in Section 1.7 will be used for the calculations presented in this section. The cross-section was selected to provide a means of demonstrating calculation procedures and does not represent any slab currently in use. Therefore, this generic slab should never be specified for use on a project. See Section 1.7 for the slabs currently available.

2.2 Flexural Design

2.2.1 ACI Requirements

Chapter 18 of ACI (318-95) presents provisions for the flexural design of prestressed concrete members. The applicable limits from ACI are paraphrased as follows:

2.2.1.1 Permissible stresses at transfer (Section 18.4).

a) Extreme fiber stress in compression . . . . $0.6\,f'_{ci}$

b) Extreme fiber stress in tension except as permitted in (c) . . . . $3\sqrt{f'_{ci}}$

c) Extreme fiber stress in tension at ends of simply supported members . . . . $6\sqrt{f'_{ci}}$

2.2.1.2 Permissible stresses at service loads (Section 18.4)

a) Extreme fiber stress in compression due to prestress plus sustained loads . . . . $0.45\,f'_c$

b) Extreme fiber stress in compression due to prestress plus total load . . . . $0.60\,f'_c$

c) Extreme fiber stress in tension in precompressed tensile zone . . . . . $6\sqrt{f'_c}$

d) Extreme fiber stress in tension in precompressed tensile zone where deflections are calculated considering bilinear moment-deflection relationships . . . . $12\sqrt{f'_c}$

2.2.1.3 Loss of prestress (Section 18.6)

Calculation of losses shall consider:

a) Seating loss
b) Elastic shortening of concrete
c) Creep of concrete
d) Shrinkage of concrete
e) Steel relaxation

2.2.1.4 Design (ultimate) strength

a) Load Factors (Section 9.2)
$U = 1.4D + 1.7L$

b) Strength Reduction Factors (Section 9.3)
Flexure $\phi = 0.9$

c) Flexural Strength (Section 18.7)

$$M_u \leq \phi M_n = \phi A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right)$$

$$a = \frac{A_{ps} f_{ps}}{0.85 f'_c b}$$

f_{ps} = value calculated by strain compatibility or

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\rho\frac{f_{pu}}{f'_c}\right)$$

$M_n > 1.2\ M_{cr}$

2.2.2 Stresses at Transfer

When the prestressing strands are cut to apply the prestressing force to the concrete, only the slab self weight is present to counteract the effects of eccentric prestress. A check of stresses is required at this point to determine the concrete strength required to preclude cracking on the tension side or crushing on the compression side. The concrete strength at the time of transfer may be only 50% to 60% of the 28 day design strength.

Example 2.2.2.1 - Transfer Stresses

Using the generic hollow core cross-section defined in Section 1.7, check stresses at transfer of prestress using the following criteria:

Prestressing steel: 4 - $^1/_2''$ dia. 270 ksi, low relaxation strands.

$A_{ps} = 4(0.153) = 0.612\ in^2$

assume 5% initial loss

$d_p = 7''$

$\ell = 30'\text{-}6''$

initial stress = 70% f_{pu}

Solution:

Stresses will be checked at the transfer point and at midspan

At release prestress force

$P_o = (0.70)(0.95)(0.612)(270) = 109.9k$

Prestress effect

$$= \frac{P_o}{A} \mp P_o\frac{e}{S}$$

$$= \frac{109.9}{154} \mp \frac{109.9(2.89)}{\begin{cases}297.9\\314.8\end{cases}}$$

= –0.353 ksi top fiber

= +1.723 ksi bottom fiber

Self weight at transfer point

$\ell t = 50d_b = 50(1/2) = 25$ in

moment 25 in from slab end

$$M_d = \left(\frac{30.5}{2}(2.08) - \frac{2.08^2}{2}\right)(0.0535)(3')$$

= 4.74 ft-k

$$\frac{M_d}{S} = \frac{(4.74)(12)}{\begin{cases}279.9\\314.8\end{cases}}$$

= +0.191 ksi top fiber

= –0.181 ksi bottom fiber

Net concrete stress at transfer point

= –0.162 ksi top fiber

= +1.542 ksi bottom fiber

Self weight at midspan

$$M_d = \frac{30.5^2}{8}(0.0535)(3') = 18.66 \text{ ft-k}$$

$$\frac{M_d}{S} = \frac{(18.66)(12)}{\begin{cases}279.9\\314.8\end{cases}}$$

= +0.752 ksi top fiber

= –0.711 ksi bottom fiber

Net concrete stress at midspan

= +0.399 ksi top fiber

= +1.012 ksi bottom fiber

Allowable stresses:

tension at end = $6\sqrt{f'_{ci}}$

$$f'_{ci} = \left(\frac{-162}{6}\right)^2 = 729 \text{ psi}$$

tension at midspan = $3\sqrt{f'_{ci}}$

does not control

compression = 0.6 f'_{ci}

$$f'_{ci} = \frac{1542}{0.6} = 2570 \text{ psi}$$

Concrete strength required at release

= 2570 psi

Note that if tension or compression in the end region exceeds allowables based on a reasonable concrete release strength, strands may be debonded in some manufacturing systems or, for tension, top mild reinforcement may be used in some manufacturing systems to resist the total tension force.

If tension in the midspan region controls, either a high release strength must be used or mild reinforcement must be added to resist the total tension force. Mild reinforcement should only be used in the wet cast manufacturing system.

2.2.3 Prestress Losses

The calculation of prestress losses affects the service load behavior of a slab. The accuracy of any calculation method is dependent on the preciseness of concrete and prestressing steel material properties as well as external factors such as humidity used in the calculation procedure. The accuracy of loss calculations has little effect on the ultimate strength of a member.

Prestress loss calculations are required for prediction of camber and for service load stress calculations. Since the success of a project is judged on service load performance rather than ultimate strength, it behooves any slab producer to use a loss calculation procedure which best predicts the behavior of the product as produced.

For low relaxation strand and for special cases (e.g., long spans or special loadings) using stress relieved strand, the 1995 ACI Code references several sources for prestress loss calculations. The method presented here was developed by Zia, et al.[5] and considers the following parameters:

1) <u>Elastic Shortening</u>

$$ES = K_{es}\frac{E_s}{E_{ci}}f_{cir}$$

K_{es} = 1.0 for pretensioned members

$$f_{cir} = K_{cir}\left(\frac{P_i}{A} + \frac{P_i e^2}{I}\right) - \frac{M_g e}{I}$$

K_{cir} = 0.9 for pretensioned members

2) <u>Concrete Creep</u>

$$CR = K_{cr}\frac{E_s}{E_c}(f_{cir} - f_{cds})$$

K_{cr} = 2.0 for normal weight pretensioned members

= 1.6 for sand lightweight pretensioned members

$$f_{cds} = \frac{M_{sd}e}{I}$$

3) <u>Shrinkage of Concrete</u>

$$SH = 8.2 \times 10^{-6}K_{sh}E_s\left(1 - 0.06\frac{V}{S}\right) \times (100 - RH)$$

Fig. 2.2.3.1 Ambient relative humidity

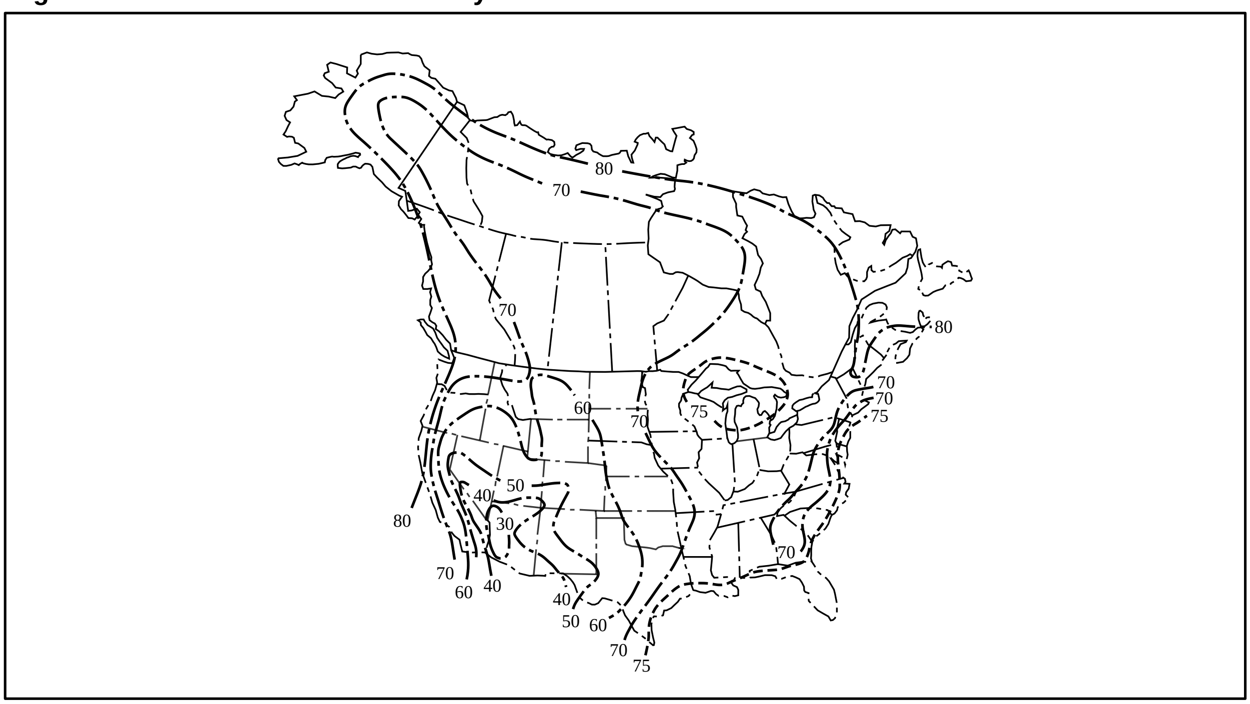


Table 2.2.3.1

Type of tendon	K_{re} psi	J
270 Grade stress-relieved strand or wire	20,000	0.15
250 Grade stress-relieved strand or wire	18,500	0.14
240 or 235 Grade stress-relieved wire	17,600	0.13
270 Grade low-relaxation strand	5000	0.040
250 Grade low-relaxation wire	4630	0.037
240 or 235 Grade low-relaxation wire	4400	0.035
145 or 160 Grade stress-relieved bar	6000	0.05

K_{sh} = 1.0 for pretensioned members

RH = Ambient relative humidity from Figure 2.2.3.1

4) Steel Relaxation

$$RE = [K_{re} - J\,(SH + CR + ES)]C$$

K_{re}, J, C = factors from Tables 2.2.3.1 and 2.2.3.2

5) Total Loss = ES + CR + SH + RE

Observations and experience in a plant may provide modifications to loss calculations to better predict slab performance.

Example 2.2.3.1 Loss of Prestress

Using the generic hollow core cross-section defined in Section 1.7, calculate the loss of prestress based on the following information:

Prestressing steel: 4-$^1/_2$″ dia. 270 ksi, low relaxation strands

$A_{ps}f_{pu}$ = 0.153(270) = 41.3k/strand

d_p = 7″

initial stress = 70% f_{pu}

ℓ = 30′-6″

Superimposed dead load = 20 psf

Table 2.2.3.2 Values of C

f_{si}/f_{pu}	Stress-relieved strand or wire	Stress-relieved bar or low-relaxation strand or wire
0.80		1.28
0.79		1.22
0.78		1.16
0.77		1.11
0.76		1.05
0.75	1.45	1.00
0.74	1.36	0.95
0.73	1.27	0.90
0.72	1.18	0.85
0.71	1.09	0.80
0.70	1.00	0.75
0.69	0.94	0.70
0.68	0.89	0.66
0.67	0.83	0.61
0.66	0.78	0.57
0.65	0.73	0.53
0.64	0.68	0.49
0.63	0.63	0.45
0.62	0.58	0.41
0.61	0.53	0.37
0.60	0.49	0.33

Solution:

1) Elastic Shortening

$$P_i = 0.7(4)(41.3k) = 115.6k$$

$$M_g = \frac{30.5^2}{8}(0.0535)(3') = 18.66 \text{ ft-k} = 224 \text{ in-k}$$

$$f_{cir} = 0.9\left(\frac{115.6}{154} + \frac{115.6(2.89)^2}{1224.5}\right) - \frac{(224)(2.89)}{1224.5} = 0.857 \text{ ksi}$$

using E_s = 28,500 ksi and E_{ci} = 3250 ksi

$$ES = K_{es}\frac{E_s}{E_{ci}}f_{cir} = (1.0)\frac{28500}{3250}(0.857)$$

$= 7.52$ ksi

2) Concrete Creep

$$f_{cds} = \frac{M_{sd}e}{I}$$

$$= \frac{\left(\frac{30.5^2}{8}\right)(0.02)(3)(12)(2.89)}{1224.5}$$

$= 0.198$ ksi

using $E_c = 4300$ ksi and normal weight concrete

$$CR = K_{cr}\frac{E_s}{E_c}(f_{cir} - f_{cds})$$

$$= (2.0)\frac{28500}{4300}(0.857 - 0.198)$$

$= 8.74$ ksi

3) Shrinkage of Concrete

$$\frac{V}{S} = \frac{\text{Area}}{\text{Perimeter}} = \frac{154}{2(36 + 8)} = 1.75$$

use RH = 70%

$$SH = 8.2 \times 10^{-6} K_{sh} E_s \left(1 - 0.06\frac{V}{S}\right) \times (100 - RH)$$

$$= 8.2 \times 10^{-6}(1.0)28500 \times (1 - 0.06 \times 1.75)(100 - 70)$$

$= 6.27$ ksi

4) Steel Relaxation

From Table 2.2.3.1

$K_{re} = 5000$, $J = 0.04$

From Table 2.2.3.2

$C = 0.75$ for $f_{si}/f_{pu} = 0.7$

$$RE = [K_{re} - J(SH + CR + ES)]C$$

$$= \left[\frac{5000}{1000} - 0.04 \times (6.27 + 8.74 + 7.52)\right]0.75$$

$= 3.07$ ksi

5) Total Loss at Midspan

$= 7.52 + 8.74 + 6.27 + 3.07$

$= 25.6$ ksi

$$\% = \frac{25.6}{(0.7)(270)}(100) = 13.5\%$$

2.2.4 Service Load Stresses

Service load concrete stresses are calculated as a measure of performance or serviceability. For the in-service state when deflections must be calculated, a stress check must first be made to determine whether gross section properties or cracked-transformed section properties are to be used.

In-service stresses are checked assuming that all prestress losses have occurred. The calculated stresses are compared to the permissible stresses noted in Section 2.2.1. Hollow core slabs are normally designed to be uncracked under full service loads. Tensile stress limits of between $6\sqrt{f'_c}$ and $7.5\sqrt{f'_c}$ are commonly used. In special circumstances where deflections will not be a problem and where cracking will not be of concern, the upper limit of $12\sqrt{f'_c}$ can be used.

Example 2.2.4.1 Service Load Stresses

Using the generic hollow core cross-section defined in Section 1.7, calculate the service load stresses given the following criteria:

Prestressing steel:

4-1/2″ dia. 270 ksi, low relaxation strands

$A_{ps}f_{pu} = 0.153(270) = 41.3$k/strand

$d_p = 7''$

Initial stress = 70% f_{pu}

$f'_c = 5000$ psi

$\ell = 30'\text{-}6''$

Clear Span = 30′-0″

Superimposed Dead Load = 20 psf

Live Load = 50 psf

<u>Solution</u>:

$$M_{sustained} = \frac{30^2}{8}(0.0535 + 0.020)$$

$= 8.27$ ft-k/ft $= 99.2$ in-k/ft

$$M_{service} = \frac{30^2}{8}(0.0535 + 0.020 + 0.050)$$

$= 13.89$ ft-k/ft $= 167$ in-k/ft

With losses = 13.5% from Example 2.2.3.1

$A_{ps}f_{se} = (0.7)(4)(41.3)(1 - 0.135)$
$= 100.0k$

Top fiber compression with sustained loads

$$f_{top} = \frac{100.0}{154} - \frac{100.0(2.89)}{297.9} + \frac{99.2(3)}{297.9}$$
$$= 0.649 - 0.970 + 0.999$$
$$= +0.679 \text{ ksi}$$

Permissible compression

$$= 0.45f'c$$
$$= 0.45(5000)$$
$$= 2.25 \text{ ksi} > 0.679 \text{ ksi} \quad \text{OK}$$

Top fiber compression with total load

$$f_{top} = \frac{100.0}{154} - \frac{100.0(2.89)}{297.9} + \frac{167(3)}{297.9}$$
$$= 0.649 - 0.970 + 1.679$$
$$= 1.358 \text{ ksi}$$

Permissible compression

$$= 0.60f'c$$
$$= 0.60(5000)$$
$$= 3.00 \text{ ksi} > 1.358 \text{ ksi} \quad \text{OK}$$

Bottom fiber tension

$$f_{bottom} = 0.649 + (0.970 - 1.679)\frac{297.9}{314.8}$$
$$= -0.022 \text{ ksi (tension)}$$

Permissible tension

$$= 7.5\sqrt{f'_c}$$
$$= 7.5\sqrt{5000}$$
$$= 0.530 \text{ ksi} > 0.022 \text{ ksi} \quad \text{OK}$$

2.2.5 Design Flexural Strength

The moment capacity of a prestressed member is a function of the ultimate stress developed in the prestressing strands. As with non-prestressed concrete, upper and lower limits are placed on the amount of reinforcing to ensure that the stress in the strands is compatible with concrete stresses for ductile behavior.

The lower limit of reinforcing requires that:

$$\phi M_n \geq 1.2\ M_{cr}$$

$$M_{cr} = \frac{I}{y_b}\left(\frac{P}{A} + \frac{Pe}{S_b} + 7.5\sqrt{f'_c}\right)$$

This ensures that when the concrete develops flexural cracks, the prestressing steel will not have reached its full design stress. Violation of this criteria might result in strand fractures at the point of flexural cracking with a resulting brittle failure. However, ACI (318-95) Section 18.8.3 allows violation of this requirement for flexural members with shear and flexural strength at least twice that required.

The upper limit of reinforcing requires that,

$$\omega_p \text{ or,}$$

$$\left[\omega_p + \frac{d}{d_p}(\omega - \omega')\right] \text{ or}$$

$$\left[\omega_{pw} + \frac{d}{d_p}(\omega_w - \omega'_w)\right]$$

be not greater than $0.36\beta_1$

The need for an upper limit on reinforcing is related to the assumptions of ultimate concrete compressive strain. Using a uniform compression stress block forces more concrete to reach ultimate strain as reinforcing ratios increase. Therefore when the upper reinforcing limit is exceeded, the moment capacity must be based on the compression block. For this condition,

$$\phi M_n = \phi\left[f'_c b d_p^2\left(0.36\beta_1 - 0.08\beta_1^2\right)\right]$$

for rectangular sections or for flanged sections with the neutral axis within the flange.

The stress in the prestressing steel at ultimate may be calculated in several ways. The ACI equation (18-3) may be used as an approximation, charts and tables from the PCI Design Handbook may be used, or a strain compatibility analysis may be made.

Example 2.2.5.1 Design Flexural Strength

Using the generic hollow core slab defined in Section 1.7, check the design flexural strength given the following criteria:

Prestressing steel: 4-1/2″ dia., 270 ksi, low relaxation strands

d_p = 7″

initial stress = 70% f_{pu}

f′c = 5000 psi

ℓ = 30′-6″

Clear span = 30′-0″

Superimposed Dead Load = 20 psf

Live Load = 50 psf

<u>Solution:</u>

<u>METHOD 1:</u> ACI Equation (18-3)

$$\phi M_n = \phi A_{ps} f_{ps}(d_p - a/2)$$

$$f_{ps} = f_{pu}\left[1 - \frac{\gamma_p}{\beta_1}\left(\rho_p \frac{f_{pu}}{f'_c}\right)\right]$$

Use $\gamma_p = 0.28$ for low relaxation strands

$$\beta_1 = 0.85 - \left(\frac{5000 - 4000}{1000}\right) 0.05$$

$$= 0.80$$

$$\rho_p = \frac{A_{ps}}{bd_p} = \frac{4(0.153)}{(36)(7)} = 0.0024$$

$$f_{ps} = 270\left[1 - \frac{0.28}{0.80}\left(0.0024\frac{270}{5}\right)\right]$$

$$= 257.7 \text{ ksi}$$

$$\omega_p = \frac{\rho_p f_{ps}}{f'_c} = \frac{0.0024(257.7)}{5}$$

$$= 0.124 < 0.36\,\beta_1 = 0.288 \quad \text{OK}$$

$$a = \frac{A_{ps} f_{ps}}{0.85 f'_c b} = \frac{4(0.153)(257.7)}{(0.85)(5)(36)}$$

$$= 1.03 \text{ in}$$

Note: If "a" exceeds the top flange thickness, the compression block will encroach on the core area. For this situation, multiple compression forces are used for the internal couple as is done with other flanged members.

$$\phi M_n = 0.9(4)(0.153)(257.7)\left(7 - \frac{1.03}{2}\right)$$

$$= 920 \text{ in-k/slab} = 76.7 \text{ ft-k/slab}$$

$$w_u = 1.4(0.0535 + 0.02) + 1.7(0.05)$$

$$= 0.188 \text{ ksf}$$

$$M_u = \frac{30^2}{8}(0.188)$$

$$= 21.14 \text{ ft-k/ft}$$

$$= 63.4 \text{ ft-k/slab} < 76.7 \quad \text{OK}$$

Check minimum reinforcement

$$\phi M_n \geq 1.2\, M_{cr}$$

From Example 2.2.3.1

Loss = 13.5%

$$A_{ps} f_{se} = 0.7(4)(41.3)(1 - 0.135)$$

$$= 100.0 \text{ k}$$

Bottom compression

$$= \frac{100.0}{154} + \frac{100.0(2.89)}{314.8}$$

$$= 1.567 \text{ ksi}$$

$$M_{cr} = \frac{1224.5}{3.89}\left(1.567 + \frac{7.5\sqrt{5000}}{1000}\right)$$

$$= 660 \text{ in-k/slab}$$

$$\frac{\phi M_n}{M_{cr}} = \frac{920}{660} = 1.39 > 1.2 \quad \text{OK}$$

<u>METHOD 2:</u> PCI Design Handbook

Using Figure 4.12.2 from the 5th Edition Handbook.

$$\omega_{pu} = \frac{A_{ps}}{bd_p}\frac{f_{pu}}{f'_c}$$

$$= \frac{4(41.3)}{(36)(7)(5)}$$

$$= 0.131$$

$$K'_u = 538$$

$$\phi M_n = K'_u \frac{bd_p^2}{12000}$$

$$= 538\left(\frac{36(7)^2}{12000}\right)$$

$$= 79.0 \text{ ft-k/slab}$$

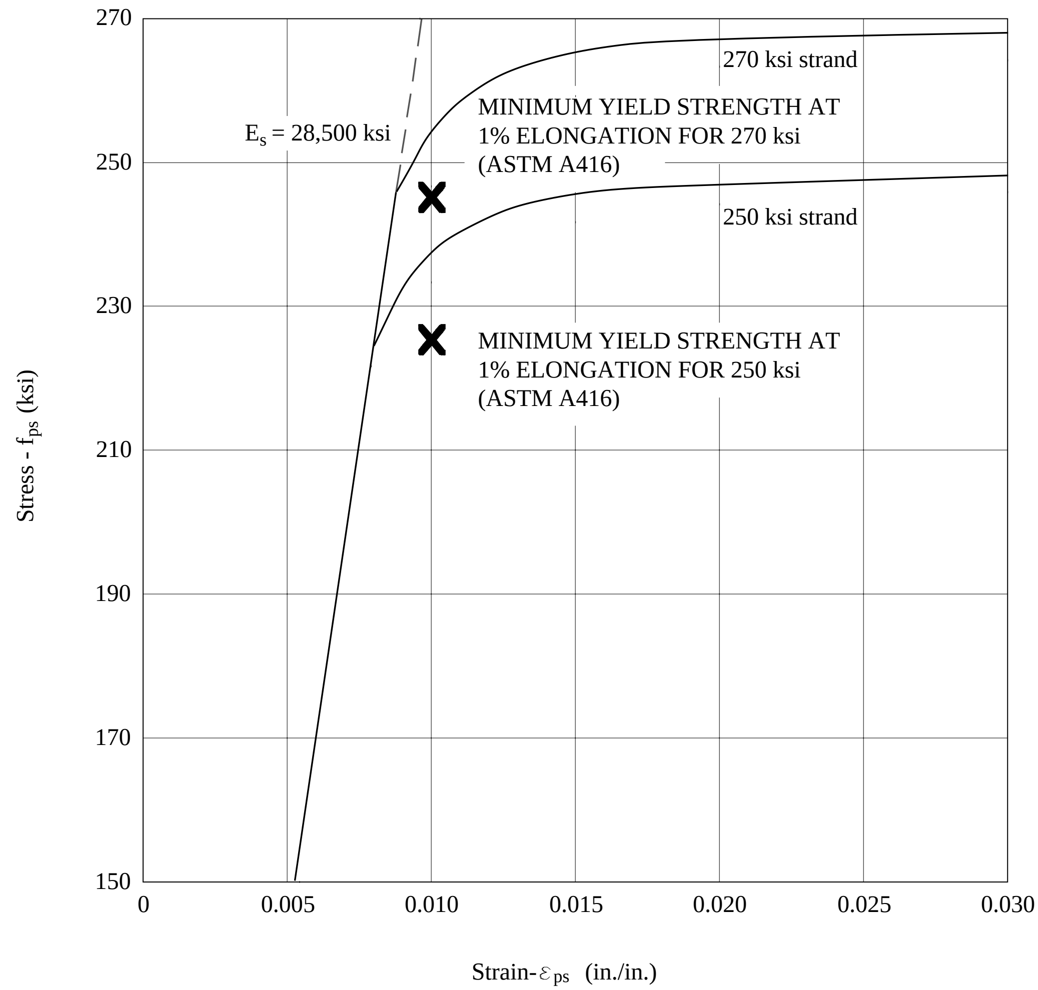
<u>METHOD 3:</u> Strain Compatibility

The stress-strain diagram from Figure 11.2.5 of the PCI Design Handbook, shown in Fig. 2.2.5.1, will be used for this example. However, the actual stress-strain curves received with strand mill reports should be used when available.

The concrete ultimate strain is assumed to be 0.003 in/in. The method involves a trial and error procedure to obtain equilibrium within the section where the force in the compression block equals the tensile force in the steel. The equations are developed from the strain diagram shown.

$$a = \beta_1 c$$

Fig. 2.2.5.1 Stress-strain curves, prestressing strand



These curves can be approximated by the following equations:

250 ksi strand

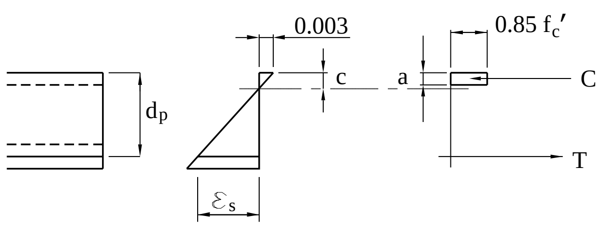
$\varepsilon_{ps} \le 0.0076$: $f_{ps} = 28{,}500\ \varepsilon_{ps}$ (ksi)

$\varepsilon_{ps} > 0.0076$: $f_{ps} = 250 - \dfrac{0.04}{\epsilon_{ps} - 0.0064}$ (ksi)

270 ksi strand

$\varepsilon_{ps} \le 0.0086$: $f_{ps} = 28{,}500\ \varepsilon_{ps}$ (ksi)

$\varepsilon_{ps} > 0.0086$: $f_{ps} = 270 - \dfrac{0.04}{\epsilon_{ps} - 0.007}$ (ksi)



Using 13.5% loss from Example 2.2.3.1

$$f_{se} = 0.7(270)(1 - 0.135) = 163.4 \text{ ksi}$$

$$\varepsilon_{se} = \frac{f_{se}}{E_s} = \frac{163.4}{28500} = 0.0057$$

Assume c = 1″ then a = 0.80(1) = 0.8″

$$\varepsilon_s = \frac{d_p}{c}(0.003) - 0.003$$

$$= \frac{7}{1}(0.003) - 0.003 = 0.018$$

$$\varepsilon_{ps} = \varepsilon_{se} + \varepsilon_s$$

$$= 0.0057 + 0.018 = 0.0237$$

From stress-strain curve

$$f_{ps} = 268 \text{ ksi}$$

$$T = 4(0.153)(268) = 163.8$$

$$C = 0.85(5)(0.8)(36)$$

$$= 122.4\text{k} < 163.8\text{k}$$

Try c = 1.3″ then a = 0.80(1.3) = 1.04″

$$\varepsilon_s = \frac{7}{1.30}(0.003) - 0.003$$

$$= 0.0131$$

$$\varepsilon_{ps} = 0.0131 + 0.0057 = 0.0188$$

From stress-strain curve

$$f_{ps} = 267 \text{ ksi}$$

$$T = 4(0.153)(267) = 163$$

$$C = 0.85(5)(1.04)(36)$$

$$= 159\text{k} \approx 163\text{k}$$

$$\phi M_n = 0.9(4)(0.153)(267)\left(7 - \frac{1.04}{2}\right)$$

$$= 952 \text{ in-k/slab} = 79.3 \text{ ft-k/slab}$$

On occasion, conventional reinforcement is added to a hollow core slab to locally provide added flexural strength. When required, the bars are placed in cores right after the slab is cast and concrete is added to fill the cores with the bars. The following example illustrates the flexural strength calculation.

Example 2.2.5.2 Flexural Strength with Bars

Repeat Example 2.2.5.1 but add 2 - #4 bars in cores.

<u>Solution:</u>

Use strain compatibility for strength calculation with an effective depth of 5.5 in for the #4 bars.

Assume c = 1.53 in.

then a = 0.80(1.53) = 1.22 in

for strands

$$\varepsilon_s = \frac{7}{1.53}(0.003) - 0.003$$

$$= 0.0107 \text{ in/in}$$

$$\varepsilon_{ps} = 0.0057 + 0.0107$$

$$= 0.0164 \text{ in/in}$$

$$f_{ps} = 266 \text{ ksi}$$

for bars

$$\varepsilon_s = \frac{5.5}{1.53}(0.003) - 0.003$$

$$= 0.0078 \text{ in/in}$$

$$\text{yield strain} = \frac{60}{29000} = 0.002 \text{ in/in}$$

$$T = 4(0.153)(266) + 2(0.2)(60)$$

$$= 162.8 + 24$$

$$= 186.8\text{k}$$

$$C = 0.85(5)(1.22)(36)$$

$$= 186.7\text{k} \cong 186.8\text{k} \quad \text{ok}$$

$$\phi M_n = 0.9\left[162.8\left(7 - \frac{1.22}{2}\right) + 24\left(5.5 - \frac{1.22}{2}\right)\right]$$

$$= 1042 \text{ in-k}$$

$$= 86.8 \text{ ft-k}$$

2.3 Shear Design

2.3.1 ACI Requirements

Hollow core slabs are designed for shear according to the same ACI Code provisions used in general for prestressed members. In dry cast systems, the normal practice is to not provide stirrups when the applied shear exceeds shear capacity because of the difficulty encountered placing stirrups in most production processes. The placement of stirrups in a wet cast system is certainly easier than in a dry cast extruded system and is a viable shear enhancement method. An alternative used to increase shear capacity is to reduce the number of cores used in a given slab. This may be done by either leaving out a core for the entire length of a slab or by locally breaking into the cores and filling them solid while the concrete is still in a somewhat plastic state.

The provisions for shear are found in Chapter 11 of ACI 318-95. With some paraphrasing, the requirements are:

$$V_u \leq \phi V_n$$

$$\phi = 0.85 \text{ for shear}$$

$$V_n = V_c + V_s$$

For the purpose of this discussion, V_s, the contribution of shear reinforcement, will be taken as zero. The nominal concrete shear strength may be found using equation (11-9),

$$V_c = \left(0.6\sqrt{f'_c} + 700\frac{V_u d}{M_u}\right) b_w d \qquad (11\text{-}9)$$

when the effective prestress force is not less than 40 percent of the tensile strength of the flexural reinforcement. The term $V_u d/M_u$ shall not exceed 1.0. The minimum value for V_c may be used as $2\sqrt{f'_c}\, b_w d$ and the maximum value is the lesser of $5\sqrt{f'_c}\, b_w d$ or the value obtained from Equation (11-12) considering reduced effective prestress in the transfer zone.

Alternatively more refined shear calculations can be made according to the lesser of Equations (11-10) or (11-12).

$$V_{ci} = 0.6\sqrt{f'_c}\, b_w d + V_d + \frac{V_i M_{cr}}{M_{max}} \qquad (11\text{-}10)$$

$$V_{cw} = (3.5\sqrt{f'_c} + 0.3 f_{pc})\, b_w d \qquad (11\text{-}12)$$

Equation (11-10) predicts shear strength for an inclined shear failure mode. For Equation (11-10), the following relationships are used:

$$M_{cr} = \left(\frac{I}{y}\right)(6\sqrt{f'_c} + f_{pe} - f_d) \qquad (11\text{-}11)$$

V_d = Unfactored self weight shear for non-composite sections

V_i = $V_u - V_d$

M_{max} = $M_u - M_d$

M_d = Unfactored self weight moment for non-composite sections

The minimum value for V_{ci} need not be less than $1.7\sqrt{f'_c}\, b_w d$ or $2\sqrt{f'_c}\, b_w d$ when the effective prestress force is not less than 40% of the tensile strength of the flexural reinforcement. For equations (11-10), (11-11) and (11-12), the reduction in prestressing force at the member end due to transfer must be considered. The ACI Code allows an assumption that prestressing force increases linearly from zero at the member end to full effective prestress in a length equal to 50 strand diameters.

Example 2.3.1.1 Shear Design

Using the generic hollow core cross-section defined in Section 1.7, check the slab for shear given the following information:

Prestressing steel: 4-1/2″ dia., 270 ksi, low relaxation strands.

Initial stress = 70% f_{pu} loss = 15%

f'_c = 5000 psi

ℓ = 25′-6″

Clear span = 25′-0″

Superimposed Dead Load = 20 psf

Live Load = 50 psf

Masonry dead load = 800 plf at 3′ from one support

<u>Solution:</u>

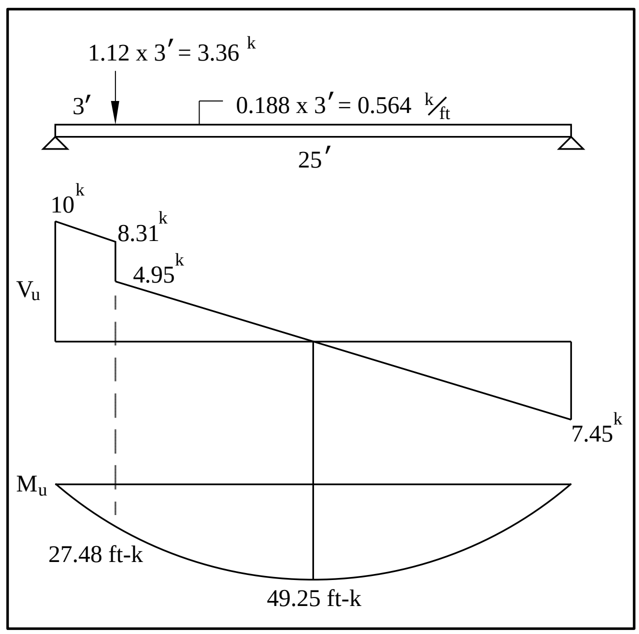
Uniform load: w_u = 1.4(0.0535 + 0.020) + 1.7(0.05)

= 0.188 ksf = 0.564 klf

Line Load: P_u = 1.4(0.800) = 1.12k/ft

= (3′)(1.12) = 3.36k

Load, shear and moment diagrams for 3′ slab width:



Using the more refined approach according to ACI Equations (11-10) or (11-12), ϕV_c is:

$$\phi V_{cw} = \frac{0.85}{1000}\left(3.5\sqrt{5000} + 0.3 f_{pc}\right) \times (10.5)(7) \qquad (11\text{-}12)$$

$$= 15.46 + 0.0187 f_{pc}$$

f_{pc} is calculated as a function of the transfer of prestress into the section along the span.

transfer length = 50 d_b = 50(1/2) = 25″
with bearing length = 3″
full prestress transfer is achieved 22″ from the face of support

$$A_{ps}f_{se} = 4(41{,}300)(0.70)(1 - 0.150) \times \left(\frac{x+3}{25}\right) \text{ to } x = 22''$$

$$f_{pc} = \frac{A_{ps}f_{se}}{A} = \frac{98294}{154}\left(\frac{x+3}{25}\right)$$

$$\phi V_{cw} = 15.46 + 0.0187\,\frac{98294}{154}\left(\frac{x+3}{25}\right)$$

$$= 15.46 + 11.96\left(\frac{x+3}{25}\right) \text{ to } x = 22''$$

$$\phi V_{ci} = \left(0.6\frac{\sqrt{5000}}{1000}(10.5)(7) + V_d + \frac{V_i M_{cr}}{M_{max}}\right) \times 0.85 \qquad (11\text{-}10)$$

V_d = Shear due to unfactored self weight (for non-composite section)

$$= 3(0.0535)\left(\frac{25}{2} - x\right) = 2.01 - 0.16x$$

V_i = Shear due to factored loads minus V_d

$$M_{cr} = \left(\frac{I}{y_b}\right)\left(6\sqrt{f'_c} + f_{pe} - f_d\right)$$

$$f_{pe} = A_{ps}f_{se}\left(\frac{1}{A} + \frac{ey_b}{I}\right)$$

$$f_{pe} = 98.294 \times \left[\frac{1}{154} + \frac{(3.89 - 1)(3.89)}{1224.5}\right]\left(\frac{x+3}{25}\right)$$

$$= 1.541\left(\frac{x+3}{25}\right) \le 1.541 \text{ ksi}$$

f_d = flexural stress due to load used for V_d

$$= \frac{M_d}{S}$$

$$= \frac{\frac{(3)(0.0535)x}{2}(25 - x)}{314.8}$$

$$= \frac{2.01x - 0.08x^2}{314.8}$$

$$M_{cr} = \frac{314.8}{12} \times \left[0.424 + f_{pe} - \frac{(2.01x - 0.08x^2)}{314.8}12\right]$$

$$= 11.130 + 26.233f_{pe} - 2.01x + 0.8x^2$$

M_{max} = Moment due to factored loads minus M_d

Based on these definitions, ϕV_{cw}, ϕV_{ci}, and V_u are calculated at intervals across the span. A summary is presented in Table 2.3.1.1. Figure 2.3.1.1 presents the results graphically.

Table 2.3.1.1 Allowable Shear

x	V_u	ϕV_{cw}	ϕV_{ci}
h/2 = 0.333′	9.82^k	18.81^k	59.40^k
0.5′	9.72	19.76	45.74
1.0′	9.44	22.64	31.92
1.5′	9.16	25.51	27.15
2.0′	8.88	27.42	23.34
2.5′	8.59	27.42	18.93
3.0′	8.31	27.42	15.98
3.0′	4.95	27.42	10.02
3.5′	4.67	27.42	9.11
4.0′	4.39	27.42	8.34

Alternatively, the simplified equation (11-9) might be used.

$$\phi V_c = 0.85\left[0.6\sqrt{5000} + 700\left(\frac{V_u}{M_u}\right)(7)\right] \times \frac{10.5(7)}{1000}$$

$$= 2.65 + 306.1\,\frac{V_u}{M_u}\ (M_u \text{ in in-k}).$$

The results of this equation are also shown on Figure 2.3.1.1.

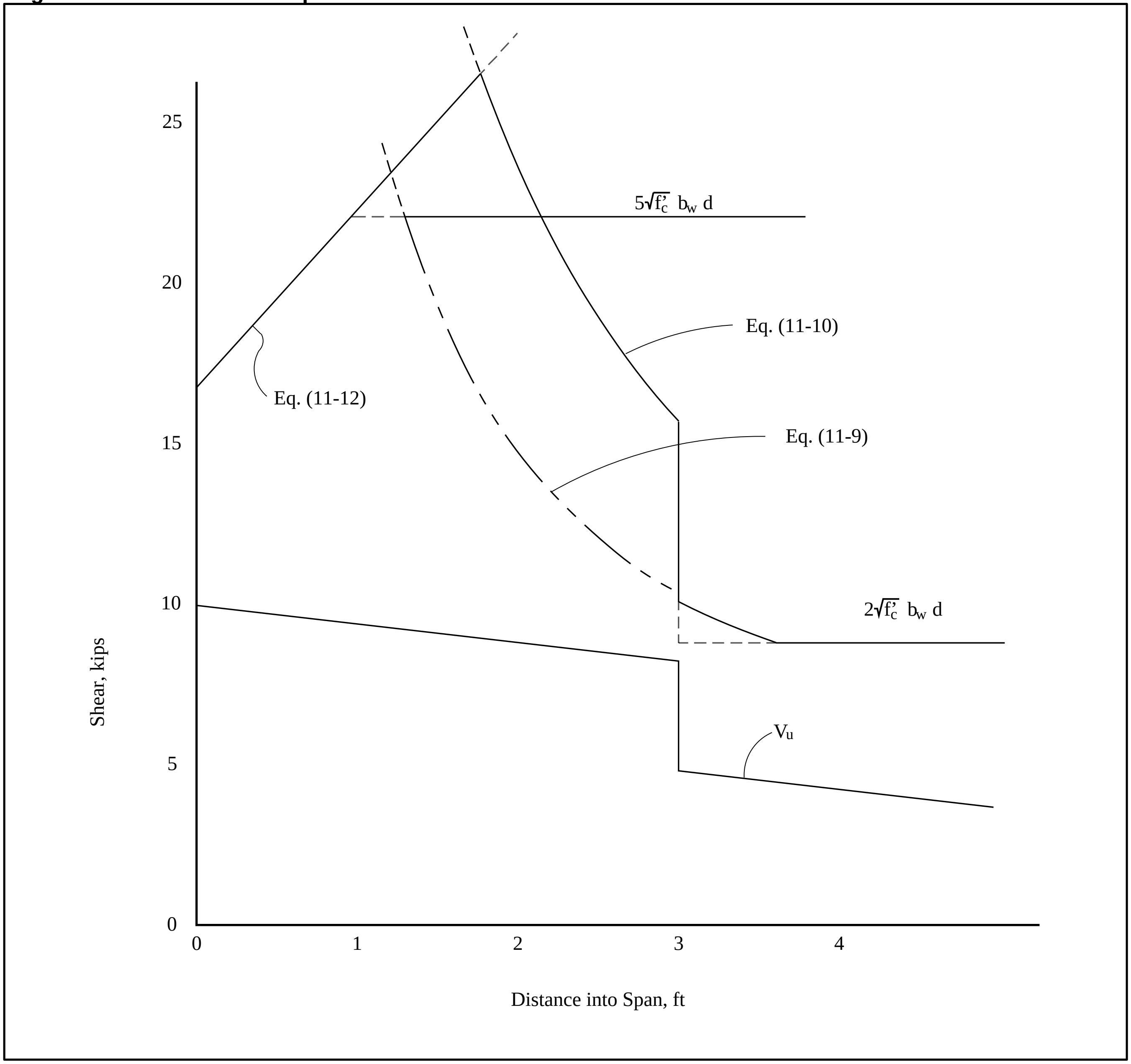
At all points, $V_u < \phi V_c$ so shear strength is adequate and stirrups are not required.

2.4 Camber and Deflection

Camber is the upward deflection of a prestressed member and results from the prestressing force being eccentric from the center of gravity of the cross-section. Since both prestressing force and eccentricity are established by the required design load and span length, camber is a result of the design rather than a design parameter. Therefore, camber requirements should not be specified.

Deflections are also affected by the amount of prestressing only because prestressing establishes the load at which a member will crack. If tensile

Fig. 2.3.1.1 Shear for Example 2.3.1.1



stresses are kept below cracking, deflections will be independent of the prestress level.

Cambers and deflections will change with time due to concrete creep, prestress loss and other factors. The sustained compression due to the prestressing will cause camber growth. Balancing this is the effect of creep on deflections due to self weight and other sustained loads. It is this time dependent movement which, in addition to instantaneous deflections, must be considered in the development of framing schemes and detailing.

Instantaneous cambers and deflections are predictable as long as the material properties are known. The time dependent cambers and deflections are not predictable with any degree of accuracy and any calculation of long term movements must be considered to be only estimates.

This section presents calculation procedures for determining long term deflections. From the producer's standpoint, history and experience must be used to modify the procedures to fit the local product. From the specifier's standpoint, these procedures will allow only approximate estimates of long term effects and should be complemented with discussions with local producers.

2.4.1 Camber

Table 2.4.1 Long term multipliers[6]

Condition	Without Composite Topping	With Composite Topping
At Erection:		
1. Deflection (downward) component - apply to the elastic deflection due to the member weight at release of prestress	1.85	1.85
2. Camber (upward) component - apply to the elastic camber due to the prestress at the time of release of prestress	1.80	1.80
Final:		
3. Deflection (downward) component - apply to the elastic deflection due to the member weight at release of prestress	2.70	2.40
4. Camber (upward) component - apply to the elastic camber due to prestress at the time of release of prestress	2.45	2.20
5. Deflection (downward) - apply to elastic deflection due to superimposed dead load only	3.00	3.00
6. Deflection (downward) - apply to elastic deflection caused by the composite topping	---	2.30

Hollow core slabs are produced with straight strand patterns rather than using draped or depressed strands. Using (+) to indicate upward movement and (–) to indicate downward movement, net camber can be calculated as:

$$\text{camber} = \frac{Pe\ell^2}{8EI} - \frac{5w\ell^4}{384EI}$$

To determine initial camber, the appropriate values for prestress force and modulus of elasticity of the concrete must be used. When ultimate moment rather than tensile stresses govern a design, the initial strand stress may be reduced to modify the anticipated camber. Additionally, slab camber is sensitive to support point locations during storage. Camber will increase as these support points move in from the slab ends.

Example 2.4.1 Initial Camber

Using the generic hollow core slab defined in section 1.7, calculate the initial camber given the following:

Prestressing steel: 4-$^1/_2''$ dia., 270 ksi, low relaxation strands

$A_{ps}f_{pu} = 0.153(270) = 41.3$k/strand

Initial stress: 70% f_{pu}

$d_p = 7''$

$\ell = 30'$-$6''$

Solution:

Estimate initial losses at 5% and use $E_{ci} = 3250$ ksi

$$P_o = 0.95(0.7)(4)(41.3) = 109.9^k$$

$$\text{camber} = \frac{109.9(3.89 - 1)[30.5(12)]^2}{(8)(3250)(1224.5)} - \frac{5(3)(0.0535)(30.5)^4(1728)}{(384)(3250)(1224.5)}$$

$$= 1.34 - 0.79$$

$$= 0.55'' \text{ Say } ^1/_2'' \text{ to } ^3/_4'' \text{ initial camber}$$

Estimating long term effects is complicated because, as time passes, the prestressing force decreases due to losses and the modulus of elasticity of the concrete increases with concrete strength gain. Traditionally, a creep factor of 2.0 has been applied to instantaneous deflections to estimate the additional deflection due to creep. This has been modified by Martin[6] for prestressed concrete. Table 2.4.1 presents suggested multipliers to determine both long term final deflections and position at erection. It should be noted that in using these multipliers, a total deflection is calculated rather than the additional increment due to long term effects.

Example 2.4.2 Long Term Camber

For the slab of Example 2.4.1, determine the net camber at erection and the final camber.

Table 2.4.2 Maximum Permissible Computed Deflections[1]

Type of member	Deflection to be considered	Deflection limitation
Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load L	$\frac{\ell^*}{180}$
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load L	$\frac{\ell}{360}$
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load)**	$\frac{\ell^{***}}{480}$
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections		$\frac{\ell^{****}}{240}$

* Limit not intended to safeguard against ponding. Ponding should be checked by suitable calculations of deflection, including added deflections due to ponded water, and considering long-term effects of all sustained loads, camber, construction tolerances, and reliability of provisions for drainage.

** Long-term deflection shall be determined in accordance with 9.5.2.5 or 9.5.4.2, but may be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be determined on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.

*** Limit may be exceeded if adequate measures are taken to prevent damage to supported or attached elements.

**** But not greater than tolerance provided for nonstructural elements. Limit may be exceeded if camber is provided so that total deflection minus camber does not exceed limit.

Solution:

At erection,

initial camber $= 1.34 - 0.79$

$= 0.55''$ from Example 2.4.1

Erection camber $= 1.34(1.80) - 0.79(1.85)$

$= 0.95''$

Say 1″ erection camber

Final camber $= 1.34(2.45) - 0.79(2.70)$

$= 1.15''$

Say approximately $1\,{}^1/_4''$ final camber

2.4.2 Deflections

As with camber, concrete creep will also affect deflections due to sustained superimposed loads. These long term effects must be considered for comparison with Table 9.5(b) of the ACI Code to determine acceptability. This table is reproduced here as Table 2.4.2. Engineering judgement should be used in comparing calculated deflections to the ACI Code limits. Many building code specified live loads exceed the actual loads in a structure. While it may be implied that the full live load be used for comparison to Table 9.5(b), situations may arise where it is more reasonable to use actual anticipated live loads for deflection comparisons. A further complication for superimposed loads is that flexural cracking will reduce the effective moment of inertia of the section. Calculations using bilinear moment-deflection relationships are required when tension exceeds $6\sqrt{f'_c}$ and are covered extensively in references 1 and 2. By definition, cracking occurs at a tensile stress of $7.5\sqrt{f'_c}$. While the ACI Code requires such bilinear calculations when $6\sqrt{f'_c}$ tension is exceeded, in effect bilinear behavior is meaningless up to a tension of $7.5\sqrt{f'_c}$. Since hollow core slabs are normally designed to be uncracked under service loads, the effects of cracking will not be considered here.

Table 2.4.1 includes multipliers for determining the long term effects for superimposed loads. Again, use of the multipliers gives an estimate of total deflection rather than an increment for the additional long term deflection.

Example 2.4.3

For the slab of Examples 2.4.1 and 2.4.2, determine the total deflection due to a superimposed load of 20 psf dead and 50 psf live on a clear span of 30′-0″ including long term effects. Use E_c = 4300 ksi.

Solution:

From Example 2.4.2

Final camber = 1.15″

superimposed dead load instantaneous deflection:

$$= \frac{5(0.02)(3)(30)^4(1728)}{(384)(4300)(1224.5)} = 0.208''$$

Final deflection = 0.208 (3.0) = 0.62″

Instantaneous live load deflection:

$$= \frac{5(0.05)(3)(30)^4(1728)}{(384)(4300)(1224.5)} = 0.52''$$

Final position

final camber	=	+ 1.15″
sustained dead load	=	– 0.62
net camber		+ 0.53″
live load increment	=	– 0.52
		+ 0.01″

For comparison to the provisions of Chapter 9 of the ACI Code, when non-structural elements are attached to the slabs, the portion of deflection after erection may be used for comparison.

Change in camber = 1.15″ – 0.95″ =	+ 0.20″
Sustained dead load =	– 0.62″
Instantaneous live loads =	– 0.52″
	– 0.94″

When a composite topping is used, it will be cast after a portion of the slab shrinkage has occurred. There will then be differential shrinkage between the topping and slab. This differential can cause additional deflection and bottom tensile stress. These effects will generally be negligible.

Example 2.4.4 Composite Slab

Given the slab of Example 2.4.3, add a 2″ composite topping and recalculate deflections including the affects of differential shrinkage.

Solution:

Final camber = 1.34 x 2.20 – 0.79 x 2.40

= 1.05″

Instantaneous topping weight deflection:

$$= \frac{5(0.025)(3)(30)^4(1728)}{(384)(4300)(1224.5)}$$

= 0.26″

Long term deflection due to topping weight

= 0.26″ (2.30) = 0.60″

Superimposed dead load deflection:

$$= \frac{5(0.02)(3)(30)^4(1728)}{(384)(4300)(2307)}$$

= 0.11″

(Note: 2307 in.^{4} = composite moment of inertia using a 3000 psi topping on a 5000 psi slab.)

Long term dead load deflection

= 0.11(3.0) = 0.33″

Instantaneous live load deflection:

$$= \frac{50}{20}(0.11) = 0.28''$$

Final Position = +1.05 – 0.60 – 0.33 – 0.26 = –0.14″ including instantaneous live load.

Calculate increment due to differential shrinkage assuming shrinkage strain of 500 × 10^{-6} in/in in both the topping and slab:

If total shrinkage	= 500 × 10^{-6}
and erection shrinkage	= 250 × 10^{-6}
differential shrinkage	= 250 × 10^{-6}

The differential shrinkage can be thought of as a prestress force from the topping where

P = $A_{topping}$ (strain)(modulus)

= 36″(2″)(0.00025)(3320)

= 59.8k

The effect is lessened by concrete creep and, using a factor of 2.30 from Table 2.4.1, reduces to:

P = 59.8/2.30 = 26k

The eccentricity of this force is:

e = 9″ – 3.89″

= 5.11″

M = Pe = 26 x 5.11 = 133 in-k

$$\text{downward deflection} = \frac{M\ell^2}{8EI}$$

$$= \frac{133(30\text{x}12)^2}{(8)(4300)(2307)}$$

= 0.22″ ≅ 1/4″

Considering the span used in this example and the accuracy of the other camber and deflection calculations, it can be easily seen that differential shrinkage will generally not be significant.

2.5 Composite Design

A composite, structural concrete topping is commonly used in floor construction with hollow core slabs. The composite action is desirable to add stiffness and strength for gravity loads and may also be required for load transfer within a diaphragm. When a composite topping is used, consideration must be given to its strength, detailing and quality assurance.

The required compressive strength of the topping may be determined from the hollow core slab design requirements. Load tables provided by local producers will normally indicate that either a 3000 psi (20.7 MPa) or 4000 psi (27.6 MPa) concrete is required. Diaphragm requirements may necessitate a higher strength topping concrete.

From a detailing standpoint, the primary consideration is that hollow core slabs will have camber. If the topping is finished as a level surface, the camber will reduce the topping thickness in the midspan region which will affect the load capacity of the slabs. With significant topping thickness reduction, the integrity of the topping concrete may also be compromised. A preliminary slab design can provide an estimate of camber and the minimum topping thickness necessary to support the design loads. The first option is to provide the minimum thickness topping at midspan and allow the thickness to increase at the slab ends to maintain a flat floor. Finish and bearing elevations can then be set to this criteria.

A second option to minimize topping concrete volume is to allow the minimum topping thickness to follow the curvature of the slabs. This will result in a finished floor with camber which may be acceptable in some occupancies. In this option, it is important that all trades be made aware of the final camber as it may affect their work. Partitions, doorways and stairs will be particularly affected in this option.

When control joints are used in a structural topping, they should be located over the joints in the precast units below where cracks would most naturally occur in the topping. At the ends of slabs, where movement will occur due to camber changes, deflections, creep, shrinkage or elastic shortening, control joints are desirable.

Reinforcing of a topping may be required for structural design. If not, consideration should be given to using minimum shrinkage reinforcement for crack control.

Since the composite topping and hollow core slabs interact to create the final structural element, it is imperative that the topping bond well with the slabs. While the building designer may only be interested in the final product, the process of achieving a well bonded, composite topping is very important. The hollow core producer is dependent on a properly bonded topping, yet is not involved in specifying, designing or installing the topping. The hollow core producer is responsible for supplying a slab that is capable of bonding with a topping. The installer of the topping is responsible for surface preparation, topping concrete mix design and curing to assure proper bond.

At a minimum, the slab surface must be clean and damp at the time of topping installation. It is recommended that the surface be thoroughly saturated prior to topping placement, but all standing water must be removed. ACI 301-96[7] specifies that a sand and cement grout be scrubbed into the slab surface ahead of topping placement. If this procedure is used, it is imperative that initial set not be allowed prior to topping placement. If initial set occurs, the grout can become a bond breaker. Similarly, bonding agents, which are rarely specified, will also act as a bond breaker if any initial set occurs prior to topping placement.

The topping concrete mix and curing techniques will also affect bond of a composite topping. Curling at topping edges or joints will cause local delamination. Curling is a result of differential shrinkage between the top and bottom surfaces of the topping. Generally, water is lost more quickly from the top surface causing additional drying shrinkage. This can be minimized by proper curing techniques and low shrinkage concrete.

Design of hollow core slabs for composite action is usually limited to a horizontal shear strength of 80 psi (0.5 MPa) according to section 17.5.2.1 of ACI 318-95. Through limited published[8] and unpublished testing, the machine finished surface has been found to meet the requirements of that section. The horizontal shear check should be based on the shear diagram rather than using an average horizontal shear over the distance from zero moment to maximum moment when checking compliance with the 80 psi limit.

Composite ties are not normally provided given the difficulty and expense of installing the ties in a machine casting operation. When the horizontal shear exceeds 80 psi (0.5 MPa) and composite ties are not used, the topping is considered to be superimposed dead load on a non-composite slab. In a wet cast system, horizontal shear ties with $^1/_4$ in amplitude roughening may be used to take advantage of the higher stresses allowed by ACI.

Design of a composite section is similar to that presented in Sections 2.2 and 2.3. The following example demonstrates the additional considerations with a composite section.

Example 2.5.1 Composite Design

Using the generic hollow core cross-section defined in Section 1.7, add a 2 in structural topping and check for the following conditions:

Prestressing steel: 4-$^1/_2''$ dia., 270 ksi low relaxation strands

Initial stress: 70% f_{pu}

d_p: 7 in

Slab: f'_c = 5000 psi

E_{ci} = 3250 ksi

E_c = 4300 ksi

Topping: f'_c = 3000 psi

E_c = 3320 ksi

Slab length: 30′-6″

Slab span: 30′-0″

Loads: topping = 25 psf
dead load = 20 psf
live load = 50 psf

Calculate section properties:

Base section $A = 154 \text{ in}^2$
$I = 1224.5 \text{ in}^4$
$y_b = 3.89$ in

Topping

$n = 3320/4300 = 0.77$

use width $= 0.77(36)$
$= 27.7$ in

Composite

$A = 154 + 2(27.7) = 209.4 \text{ in}^2$

$$y_b = \frac{154(3.89) + 2(27.7)(9)}{209.4}$$

$= 5.24$ in.

$$I = 1224.5 + 154(5.24 - 3.89)^2 + \frac{2^3}{12}(27.7) + 2(27.7)(9 - 5.24)^2 = 2307 \text{ in}^4$$

Calculate prestress losses:

From Example 2.2.3.1

ES = 7.52 ksi

Concrete creep

$$M_{sd} = \frac{30^2}{8}(0.025 + 0.020)(3) = 15.19 \text{ ft-k}$$

$$f_{cds} = \frac{15.19(12)(2.89)}{1224.5} = 0.430 \text{ ksi}$$

$$CR = (2.0)\frac{28500}{4300}(0.857 - 0.430) = 5.66 \text{ ksi}$$

SH = 6.27 ksi

$$RE = \left[\frac{5000}{1000} - 0.04(6.27 + 5.66 + 7.52)\right]0.75 = 3.17 \text{ ksi}$$

$$\text{Loss} = 7.52 + 5.66 + 6.27 + 3.17 = 22.62 \text{ ksi} = 12\%$$

Calculate service load stresses:

$$A_{ps}f_{se} = 0.7(4)(41.3)(1 - 0.12) = 101.8\text{k}$$

$$M_{non-comp} = \frac{30^2}{8}(0.0535 + 0.025) = 8.83 \text{ ft-k/ft} = 106 \text{ in-k/ft}$$

$$M_{comp} = \frac{30^2}{8}(0.020 + 0.050) = 7.88 \text{ ft-k/ft} = 94.5 \text{ in-k/ft}$$

At top of topping

$$f_{top} = \frac{94.5(3)(10 - 5.24)}{2307}(0.77) = 0.450 \text{ ksi}$$

At top of slab

$$f_{top} = \frac{101.8}{154} - \frac{101.8(2.89)(4.11)}{1224.5} + \frac{106(3)(4.11)}{1224.5} + \frac{94.5(3)(8 - 5.24)}{2307} = 1.080 \text{ ksi}$$

At bottom of slab

$$f_{bottom} = \frac{101.8}{154} + \frac{101.8(2.89)(3.89)}{1224.5} - \frac{106(3)(3.89)}{1224.5} - \frac{94.5(3)(5.24)}{2307}$$

$$= -0.058 \text{ ksi}$$

Calculate flexural strength

$$w_u = 1.4(0.0535 + 0.025 + 0.020) + 1.7(0.050)$$

$$= 0.223 \text{ ksf}$$

$$M_u = \frac{30^2}{8}(0.223)(3)$$

$$= 75.26 \text{ ft-k}$$

Using ACI Eq. (18-3)

$$\rho_p = \frac{4(0.153)}{36(9)} = 0.0019$$

$$f_{ps} = 270\left[1 - \frac{0.28}{0.85}\left(0.0019\frac{270}{3}\right)\right]$$

$$= 254.8 \text{ ksi}$$

$$a = \frac{4(0.153)(254.8)}{0.85(3)(36)}$$

$$= 1.7 \text{ in}$$

$$\phi M_n = 0.9(4)(0.153)(254.8)\left(9 - \frac{1.7}{2}\right)$$

$$= 1144 \text{ in-k} = 95.3 \text{ ft-k}$$

Check 1.2 M_{cr}

$$f_{bottom} = \frac{101.8}{154} + \frac{101.8(2.89)(3.89)}{1224.5}$$

$$= 1.596 \text{ ksi}$$

$$M_{cr} = \frac{2307}{5.24}\left(1.596 + \frac{7.5\sqrt{5000}}{1000}\right)$$

$$= 936 \text{ in-k}$$

$$\frac{\phi M_n}{M_{cr}} = \frac{1144}{936} = 1.22 > 1.2 \quad \text{ok}$$

Check horizontal shear:

$$\phi V_{nh} = \phi 80 b_v d$$

$$= 0.85(80)(36)(9)$$

$$= 22030 \text{ lb}$$

$$= 22 \text{ k}$$

at h/2

$$V_u = \left(\frac{30}{2} - \frac{10}{2(12)}\right)(0.223)(3)$$

$$= 9.8 \text{ k} < 22 \text{ k} \quad \text{ok}$$

Section is composite

Check web shear at h/2:

transfer length = 50(0.5) = 25 in

at h/2 plus 3 in bearing

$$A_{ps}f_{se} = 101.8\left(\frac{8}{25}\right) = 32.6 \text{ k}$$

for composite section, f_{pc} is calculated at centroid of composite section

$$f_{pc} = \frac{32.6}{154} - \frac{32.6(2.89)(5.24 - 3.89)}{1224.5}$$

$$= 0.108 \text{ ksi}$$

$$\phi V_{cw} = 0.85\left[\frac{3.5\sqrt{5000}}{1000} + 0.3(0.108)\right](10.5)(9)$$

$$= 22.5\text{k} > 9.8 \text{ k} \quad \text{ok}$$

Check inclined shear at 4 ft

$$V_u = \left(\frac{30}{2} - 4\right)(0.223)(3)$$

$$= 7.36\text{k}$$

$$V_d = \left(\frac{30}{2} - 4\right)(0.0535 + 0.025 + 0.020)(3)$$

$$= 3.25\text{k}$$

$$V_i = 7.36 - 3.25 = 4.11\text{k}$$

$$M_u = 0.223(3)(4)\left(\frac{30}{2} - \frac{4}{2}\right) = 34.8 \text{ ft-k}$$

$$M_d = (0.0535 + 0.025 + 0.020)(3)(4)\left(\frac{30}{2} - \frac{4}{2}\right)$$

$$= 12.25 + 3.12 = 15.37 \text{ ft-k}$$

$$M_{max} = 34.8 - 15.37 = 19.43 \text{ ft-k}$$

$$f_{pe} = \frac{101.8}{154} + \frac{101.8(2.89)(3.89)}{1224.5}$$

$$= 1.596 \text{ ksi}$$

$$f_d = \frac{12.25(12)(3.89)}{1224.5} + \frac{3.12(12)(5.24)}{2307}$$

$$= 0.552 \text{ ksi}$$

$$M_{cr} = \frac{2307}{5.24}\left(\frac{6\sqrt{5000}}{1000} + 1.596 - 0.552\right)$$

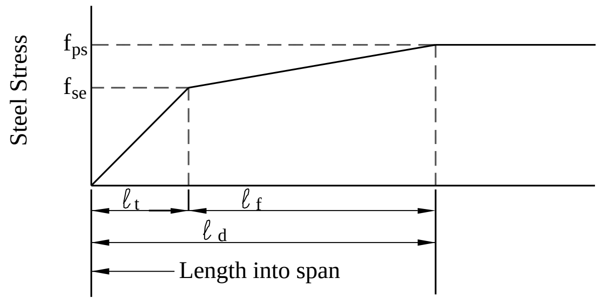
$$= 646 \text{ in-k} = 53.9 \text{ ft-k}$$

$$\phi V_{ci} = 0.85\left[\frac{0.6\sqrt{5000}}{1000}(10.5)(9)\right]$$

$$+ 0.85\left[3.25 + \frac{4.11(53.9)}{19.43}\right]$$

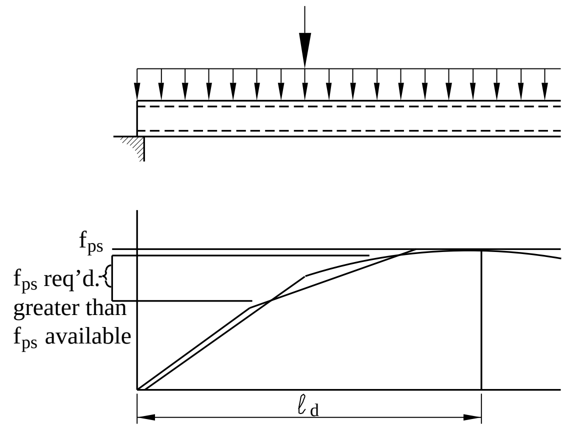
$$= 15.86\text{k} > 7.36\text{k} \quad \text{ok}$$

Fig. 2.6.1.1



Strand Development

Fig. 2.6.1.2



2.6 Strand Development

2.6.1 ACI Requirements

Section 12.9 of the ACI Code covers development length for prestressing strands. While the topic has received considerable discussion[9-16], the ACI Code expression currently remains:

$$\ell_d = (f_{ps} - 2/3 f_{se})d_b$$

A further requirement is that the development length shall be doubled when bonding of a strand does not extend to the end of the member and the precompressed tensile zone is allowed to be in tension at service loads.

The ACI Code expression for development length describes two bond mechanisms. The first is the transfer length which is the bond length required to transfer the effective prestress after losses, f_{se}, to the concrete. This portion of the development length is:

$$\ell_t = \frac{f_{se}}{3} d_b$$

With f_{se} equal to 150 ksi (1034 MPa), the transfer length becomes $50d_b$, the length used for shear calculations.

The second mechanism is for bond length after the steel stress increases above f_{se}. To develop the full design strength of the strand, f_{ps}, a bond length in addition to the transfer length is required. The flexural bond length is expressed as:

$$\ell_f = (f_{ps} - f_{se})d_b$$

Figure 2.6.1.1 depicts the increase in steel stress along the development length of the strand.

Section 12.9.2 of the ACI Code limits investigation of development length to the section nearest the end of the member where full design strength is required. In conventionally reinforced concrete, the rate of moment increase must be considered in selecting reinforcing bar sizes. This consideration is also valid in prestressed concrete members. As shown in Figure 2.6.1.2, with a steep rate of moment increase, critical sections may occur in the strand development length at less than maximum moment.

Demand on strand strength above f_{se} does not occur until after flexural cracking occurs. If flexural cracking occurs in the transfer length, the strand cannot accept additional stress so bond failure occurs. Therefore, the limit on member flexural strength in the strand transfer length is the cracking moment.

In the flexural bond length, strand stress can increase above f_{se}, but not to full f_{ps}. Therefore, there is additional flexural strength above the cracking moment, but less than full nominal strength. If flexural cracking occurs at factored load in the flexural bond length, the maximum value for f_{ps} can be calculated as:

$$f'_{ps} = f_{se} + \frac{(x - \ell_t)}{\ell_f}(f_{ps} - f_{se})$$

where x = the distance from the end of the member to the section of interest

The nominal moment capacity is then calculated on the basis of this maximum strand stress.

Martin and Korkosz[17] suggest that with partially developed strand, the full concrete compressive failure strain will not be achieved. A strain compatibility analysis can be performed to determine the concrete strain that would be consistent with f'_{ps} and nominal strength can then be calculated using that strain.

When debonded strands are mixed with fully bonded strands, a similar strain compatibility analysis may be required in the flexural bond length for the debonded strands. In this case, nominal strength can be calculated in two ways:

1. Analyze section with all strands at the f'_{ps} for the debonded strands.
2. Analyze section with only fully bonded strands at their f_{ps} and ignore the debonded strands.

The greater of the two results would predict the nominal strength of the section.

For hollow core slabs, the strain compatibility analysis for partially developed strand will yield variable results as compared to a traditional approach where f'_{ps} is used with a full concrete strain of 0.003 in/in. If f'_{ps} is close to f_{se}, the strain compatibility analysis will predict moment capacity of about 85% of the traditional analysis. When f'_{ps} is 10% greater than f_{se}, the difference reduces to 5% or less. The additional complexity of the strain compatibility analysis would only seem warranted when flexural cracking is expected near the transfer point or when debonded strands are used.

There are several aspects of a bond length discussion that are significant to hollow core slab design. In many framing schemes, there will be a requirement to use very short slabs to fill in an area. With fully developed strands, these slabs will normally have very large load capacities. However, capacity may be reduced because the strands might only be partially developed. For example, for a slab prestressed with $^1/_2''$ (12.7 mm) ϕ, 270 ksi (1860 MPa) strands with f_{se} = 150 ksi (1034 MPa) and f_{ps} = 260 ksi (1790 MPa):

$$\ell_d = \left(f_{ps} - \frac{2}{3}f_{se}\right)d_b$$

$$= \left[260 - \frac{2}{3}(150)\right](0.5)$$

$$= 80'' = 6'\text{-}8''\ (2030\ \text{mm})$$

This slab would have to be two development lengths, or 13′-4″ (4.1 m) long in order to develop its full design strength. A shorter slab would have reduced capacity.

Hollow core slab systems are often required to carry concentrated or wall loads which may affect the rate of moment increase near the member end. While not required by ACI, it is suggested that the transfer length and flexural bond length regions be investigated for reduced capacity when the moment gradient is high.

The development length equations in the ACI Code are based on testing conducted with members cast with concrete having normal water-cement ratios. As noted in the Commentary to the ACI Code, no slump concrete requires extra precautions. Hollow core slabs produced with the extrusion process fall into this category. As originally presented by Anderson and Anderson[10] and reinforced by Brooks, Gerstle and Logan[18], a measure of satisfactory bond is the free end slip of a member after it is cut to length. A limit on free end slip expressed as:

$$\delta_{all} = \frac{f_{se}f_{si}}{6E_s}d_b$$

has been suggested as a maximum free end strand slip for using the ACI Code development lengths. This expression approximates the strand shortening that would have to occur over the transfer length. For a $^1/_2''$ (12.7 mm) dia. strand stressed initially to 189 ksi (1300 MPa), the free end slip should not exceed about $^3/_{32}''$ (2.4 mm) if the ACI Code transfer and development lengths are to be used.

When free end slip exceeds δ_{all}, the transfer length and the flexural bond length will increase. Shear strength in the transfer length and moment capacity in the flexural bond length will be decreased and the length into the span where full moment capacity is provided will be increased.

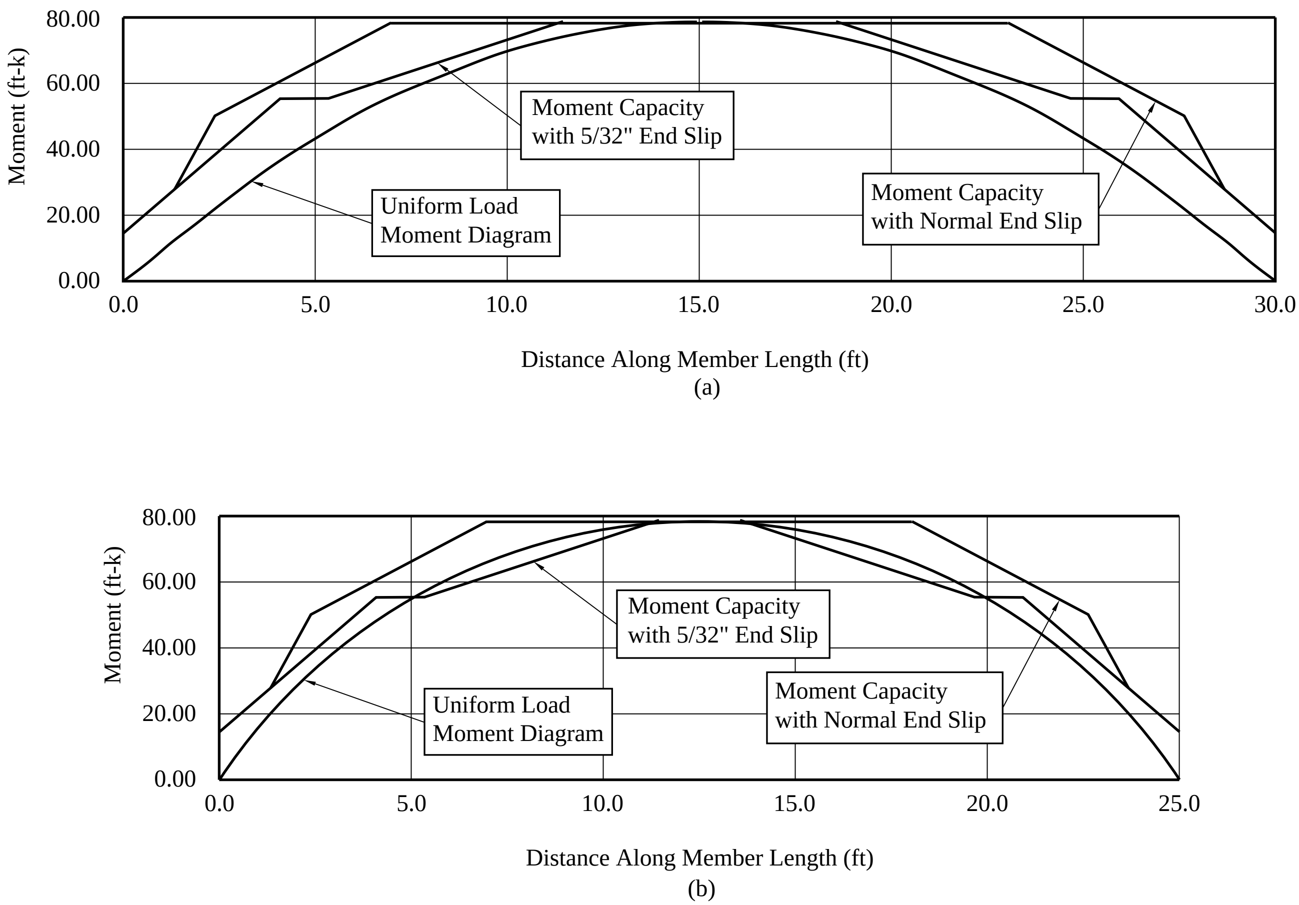
If the free end strand slip is known from quality control measurements, the member capacity can be evaluated with consideration of extended transfer and flexural bond lengths. As a function of measured end slip, the transfer length and flexural bond length can be calculated for each strand as follows:

$$\ell_t = 2\delta_s E_s/f_{si}$$

$$\ell_f = 6\delta_s E_s(f_{ps} - f_{se})/(f_{si}f_{se})$$

Shear strength can be evaluated by substituting the extended transfer length for 50 d_b in evaluating the rate of increase of prestress. Flexural

Fig. 2.6.1.3 Effect of End Slip



strength calculations are affected only by the extension of the strand development length and potential reduction of f'_{ps}. The strain compatibility analysis suggested by Martin and Korkosz for sections with partially developed strand becomes more complex as there can be variation in development lengths within a given member.

Figure 2.6.1.3 illustrates the change in moment capacity for the generic slab of Section 1.7 from normal slip to $^5/_{32}$ in (4 mm) slip on all strands. In (a), the span length is 30 ft (9.1 m) and there would be no change in slab capacity for uniform load. In (b), the span is reduced to 25 ft (7.6 m) and it is clear that the extended development length would result in reduced capacity even with uniform load. End slip in excess of normal slip has a more significant effect in shorter slabs.

The following example demonstrates the use of the Martin and Korkosz strain compatibility analysis for partially developed strand and the use of free end slip for evaluating strength. The procedure illustrated is also valid with normal end slip by using the appropriate transfer and bond lengths.

Example 2.6.1.1 Initial Strand Slip

Given the generic hollow core slab defined in Section 1.7, calculate the design flexural strength given the following:

Prestressing steel: 4-$^1/_2''$ dia., 270 ksi low relaxation strands.

E_s = 28500 ksi

d_p = 7″

f'_c = 5000 psi

f_{si} = 185 ksi
f_{se} = 163.4 ksi
f_{ps} = 267 ksi
δ_s = $^3/_{16}$ in. all strands

<u>Solution:</u>

$\ell_t = 2(^3/_{16})(28500)/185$
$= 57.8''$

$\ell_f = 6(^3/_{16})(28500)(267 - 163.4)/185/163.4$
$= 109.9''$

$\ell_d = 57.8 + 109.9$
$= 167.7''$

The minimum slab length required to achieve full flexural capacity is 2(167.7)/12 or 28 ft. Calculate flexural capacity at 10 ft.

$$f'_{ps} = 163.4 + \frac{(10x12 - 57.8)}{109.9}(267 - 163.4)$$
$$= 222 \text{ ksi}$$

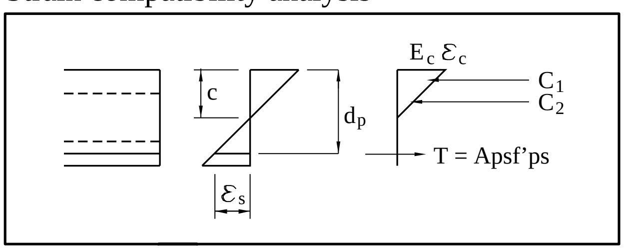
$A_{ps}f'_{ps} = 4(0.153)(222)$
$= 135.9\text{k}$

Traditional analysis

$$a = \frac{135.9}{.85(5)(36)} = 0.89 \text{ in.}$$

$M_n = 135.9(7 - 0.89/2)/12$
$= 74.24$ ft-k

Strain compatibility analysis



$\varepsilon_{ps} = \varepsilon_{se} + \varepsilon_s$
$\varepsilon_{se} = 163.4/28500$
$= 0.00573$ in/in
$\varepsilon_{ps} = 222/28500$
$= 0.00779$ in/in
$\varepsilon_s = 0.00779 - 0.00573$
$= 0.00206$ in/in

Using trial and error for

T = C

Find

c = 2.18″
ε_c = 0.000929 in/in

Concrete stress at top

= 4300(0.000929)
= 3.995 ksi

Concrete stress at top of core

$$= \frac{(2.18 - 1.25)}{2.18}(3.995) = 1.704 \text{ ksi}$$

$$C_1 = \frac{(3.984 + 1.704)}{2}(1.25)(36)$$
$$= 128\text{k}$$

$$C_2 = \frac{1.704}{2}(10.5)(2.18 - 1.25)$$
$$= 8.3\text{k}$$

$M_n = (135.9(7 - 0.54) - 8.3(1.56 - 0.54))/12$
= 72.45 ft-k